

Quantum physics

High school-adapted curriculum

Teacher's Guide

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Experimental edition

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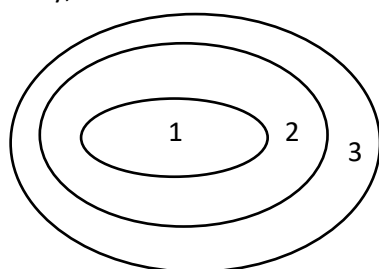
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This book aims to assist teachers presenting the basics of quantum physics in high school and offers a modular curriculum. The scope of the proposed material is aimed at about 60 hours, but it is structured in a way that allows instruction in a flexible number of hours, from a two-hour general lecture, to full instruction of all the content. This book is a teacher's guide. Therefore, the language of reference is to the teacher, and the recommendations are from the experience we gained in running the program for several years in high schools in the Physics 5 units major. Quantitative familiarity with this content in waves is required, although the program presented here does not give up solving quantitative problems. Also the mathematical level required is in tune with high-school knowledge (in particular, no knowledge of complex numbers is required).

In places where the marking appears 🍓 This means that there is activity in the raspberry, and a brief description of the task will appear.

Epistemological introduction

The innovation in this curriculum, beyond the content, is in the organization of knowledge. We take an approach of disciplinary-cultural teaching developed by Prof. Yigal Galili and Dr. Michael Zeitlin of the Hebrew University. This approach organizes knowledge in every physical theory according to three areas: nucleus, body and periphery. The nucleus includes the basic principles, the body includes all applications and examples, while the periphery is everything that confronts the theory, such as cases that are not within the jurisdiction of the theory, misunderstandings and



Disciplinary structure-Cultural of Basic Theory in Physics:

1. nucleus, The core of the theory
2. The body of knowledge of the theory
3. Periphery of the theory

the like.

Students should be introduced to this approach. A good example of this is in mechanics:

nucleus: Newton's laws.

body: All calculations resulting from Newton's laws, momentum, Energy circular motion and more.

periphery: Aristotle's conception (force creates speed and does not change it), deviations at high speeds (effects of relativity) and more.

It is worth emphasizing that since quantum physics is very unintuitive, and it undermines our simple perceptions, we will present things during the teaching in peripheral-nuclear-body order. We will first present what makes sense to us (classical understanding, which is the periphery of quantum physics), then we will present the basic principles of quantum physics, and how they confront our previous understandings, and finally we will come to applications.

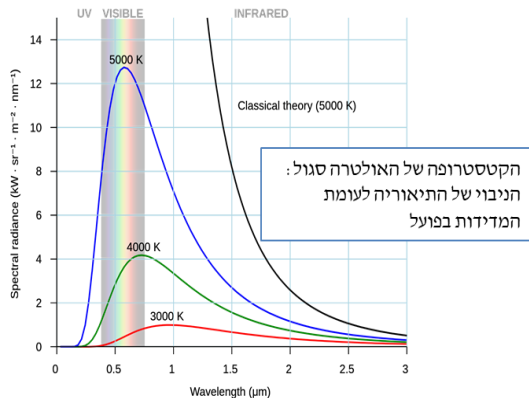
Chapter 1 - Historical Background

Most of the topics in this chapter exist in the regular curriculum, so we will not expand on them here. It is worth emphasizing that all the phenomena and conclusions in this chapter are on the periphery of classical physics, as they cannot be explained in a plausible classical way (which ultimately led to the development of quantum physics). On the other hand, these topics are also

on the periphery of quantum physics, since the explanations for them do not yet contain quantum understanding.

light - wave or particle?

In the 17th century, two major theories were formulated that attempted to describe light. Newton claimed that light is made up of particles, while Huygens argued that these are waves. About a hundred years later it was decided that the truth is with Huygens' method: that light is a wave. Phenomena such as interference and diffraction have proven this.



At the end of the 19th century physicists felt that they understood almost all physics. In 1900 the British physicist Lord Calvin declared that "the sky of physics is brighter than ever, except for two light clouds obscuring the brightness of the sky." One of those clouds was the inability of physicists to predict thermal radiation (black body radiation). This is radiation emitted by a heated body (for example, bleached iron), where the frequency of the radiation is related to the temperature, and not to the properties of the material. The theory held by the world of science failed to predict the actual results: while the long wavelengths had some degree of agreement with the theory, the measurements at short wavelengths did not match the theory, and this dramatic discrepancy was therefore dubbed the "ultraviolet catastrophe."

Physicist Max Planck gave an innovative explanation, which was an opening to a new world of physics that was not available to us until then. He explained that the energy of light comes in discrete, quantized rations. Given a particular frequency f of the light, the smallest amount of energy that can arrive ("a photon") is $E = hf$. The constant h is a fixed value called "Planck's constant". This explanation restored the particle understanding of light. This claim, which says that light comes in packets is called the quantization of light. $h = 6.626 \cdot 10^{-34} \text{ J} \cdot \text{s}$

In addition, in the late 19th century the photoelectric effect was discovered, in which the illumination of metal with light caused the emission of electrons. Some experimental facts were observed:

- i. The intensity of illumination affected the amount of electrons emitted, not their speed.
- ii. The frequency of illumination affects the speed of the electrons emitted, but not their number.
- iii. There is a threshold frequency below which there is no emission.
- iv. The emission is immediate.

Einstein explained this (using Planck's hypothesis) according to the particle principle, according to which light is composed of particles - photons, whose energy depends on their frequency, so when there is a higher frequency there is more energy in each photon-electron collision, so the electron

has more speed. The more power there is, ie, more photons, then more electrons will be emitted. For this Einstein won the Nobel Prize in 1921 (the only one he won).

It should be noted that this understanding was considered a direct proof of the quantization of light, and so it has been discussed historically. However, as we understand it today, this is not unequivocal proof, and the phenomenon can also be explained in another way.¹

In addition, in the Compton effect it is seen that in a collision between a photon and an electron it is observed that the photon has momentum. It should be noted that even at the classical level of Maxwell's equations there is momentum for radiation. The phenomena described by Huygens and the photoelectric effect and the Compton effect indicate that light is dual - both wavy and particle-like.

Chapter 2 - The Basic Principles of Quantum Physics

As mentioned, Planck's explanation of black body radiation, Einstein's explanation of the photoelectric effect and the Compton effect led to the understanding that light can be not only a wave but also a particle. A more correct formulation is to say that light has wavy properties (which are manifested in phenomena such as interference and diffraction), but in addition it has particle properties that are manifested in other cases. The relationship $p = \frac{h}{\lambda}$ (connecting the momentum p and the wavelength λ) gives the relationship between the particle properties (momentum) and those of the waves (wavelength). This feature is called the duality of light, and is sometimes called the de Broglie principle.



לואי דה-ברויי 1892-1987

Louis de Broglie, a young French duke, hypothesized that if light is not only a wave but also a particle, it is likely that matter is not only a particle but also a wave. The hypothesis was based solely on an attempt to see nature as symmetrical and beautiful. He submitted this hypothesis as a doctoral dissertation in 1923. The hypothesis was met with doubt, but Einstein was enthusiastic about it, which earned de Broglie a doctorate, and later a Nobel Prize (the first given for a doctoral dissertation).

Just as the connection given between the particle properties of light and those of the waves, here too de Broglie gave the same quantitative connection between the properties:

$$\lambda = \frac{h}{p}$$



Raspberry Activity: Developing the Photon Motion Equation Step by Step.

A few years later it was discovered that particles, such as electrons, do have wavy properties, and they perform interference and deflection like waves. We will expand on this later.

¹Lamb, WE (1995). Anti-photon. Applied physics B, 60 (2-3), 77-84

Treating a particle or body as a wave seems strange to us. Billiard balls are not waves, nor are we waves. How can we say we have a wavelength? What does that wavelength mean? We will try to understand a little more through the exercise before us:

exercise

- I. Calculate the wavelengths of the following bodies:
 1. A guy with a mass of 60 kg running at a speed of 4 meters per second.
 2. An electron moves in a circular motion around a proton, within a radius of $0.52\text{\AA} = 5.2 \cdot 10^{-11}\text{m}$.

Instruction to Section 2: Calculate the electric force and compare it to the radial force of the circular motion.
- II. Try to understand the meaning of wavelength in each case: is it noticeable? A very small wavelength is not felt in everyday life and therefore it is a feature that we cannot normally encounter. For example, the wavelength of light is very small for us, so we do not feel it as a wave usually. This is why it took centuries to be convinced that light is indeed a wave. In contrast, a wave of water or a wave in a string come in wavelengths of a noticeable magnitude for us. So we want to ask is the wavelength of the guy significant for him? Is the wavelength of the electron "significant" for its dynamics?

Case Solution 1: $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626 \cdot 10^{-34}}{60 \cdot 4} = 2.76 \cdot 10^{-36}\text{m}$

Case Solution 2: $\frac{mv^2}{r} = \frac{ke^2}{r^2} \Rightarrow v = \sqrt{\frac{k}{rm}} \cdot e \Rightarrow \lambda = \frac{h}{p} = \frac{h}{mv} = \sqrt{\frac{r}{mk}} \cdot \frac{h}{e} = 3.29\text{\AA} \cong 2\pi r$

The calculation shows that the wavelength is not noticeable for the guy, which means that quantum physics is not something we encounter in bodies with a macroscopic order of magnitude. On the other hand, for the electron it turns out that the wavelength is equal to the circumference of the circle on which it moves $2\pi r$, i.e. a magnitude that is "significant" on the scale on which the electron moves.

This observation of the particle as a wave, or as having wavy properties, is the gateway to quantum physics. Hence, we would like to understand the meaning of those waves.



If we look at a wave of matter, such as a water wave, it seems that the wave means that the matter is not concentrated at one point as a particle, but is scattered over an entire region. The function that describes the height of the wave everywhere - called the wave function, actually describes the distribution of matter, how much of the matter is at each and every point. When we claim that each particle is in fact also a wave, we are ostensibly claiming that it is not at a particular point, but is "smeared" in a particular area, and in fact is not a point particle but a cloud of matter extending in different regions at different densities, described by the wave function. In fact, this is how they understood the meaning of the waves in the early stages of the development of quantum theory.

However, there is a problem with this understanding. If we take an electron as an example, in every measurement we measure the place of the electron we find the whole electron in one place as a point particle. We do not find a part of the electron as we would expect if the electron were a cloud of matter spread in a particular region.

The perceptual transition from classical matter waves to quantum matter waves is the transition from the distribution of matter to the distribution of probability. Unlike a classical material wave, such as a water wave, whose wave function means the distribution of matter, the quantum wave function means the distribution of probability. The meaning of a quantum wave is probabilistic, that is, different probabilities are given to situations that are classically contradictory. If the classical wave of matter "allows" matter to be in several places, in different quantities, the quantum wave of matter allows the individual particle to be in several places or in several states with different probabilities. This is the quantum wave.

The wavefunction does not describe where there is more of the particle and where there is less, but rather it describes the probability of finding the particle at each and every point. The particle is a point particle, and if we measure its place it is found that it (the whole) is at a certain point, but that it has different probabilities of being in different places. If the wave function of the material wave describes the distribution of matter, then the quantum wave function describes the distribution of the probability of finding the particle everywhere.

The significance of the probability is expressed as soon as a measurement is made. If we measure the position of the particle, we can get as a result (correct!) any of the places where the wave function exists (and does not go to zero). The wave function (actually, the square of the wave function) gives the probability of getting one position or another in the measurement.



Einstein And Bohr

Einstein objected to this understanding, arguing that "God does not play dice." He disagreed with an approach that describes nature and science in a way that is purely probabilistic. To this Niels Bohr replied that he should not tell God how to act. Today experiments and evidence often show that Einstein

was wrong in this regard.

We will now expand a little on the meaning of this. In terms of classical physics, and this is how we intuitively understand reality, a physical quantity related to a particular particle or system has a single value. A classical particle has a certain location, a certain momentum, a certain energy and more. In the transition to quantum physics we understand that the particle is a wave. What is the meaning of the phrase "the particle is a wave"? If we previously defined the waves as the ability of the particle to be in different places at different probabilities, we can now say this about any feature or condition that characterizes the particle, and not just about the location feature. A particle can have several motions at once (or energy or any other physical quantity), where each value and value of the momentum has a certain probability / from now on, the waves will not necessarily look like a wave we are used to seeing (water wave, sinusoidal electromagnetic wave, etc.). In space. The perception of the waves is the understanding that the particle is not only local with defined properties, but is in a superposition that contains several possible values simultaneously. Although, Sometimes we can see that the quantum waves are manifested in classic wave phenomena, such as interference and diffraction. Thus it appears that the electron

which is a particle will present wavy phenomena as interference and diffraction. Moreover, it turns out that the phenomena can also be described as having two parts of the same entity, and these two parts are always adjacent: one part is the particle and one part is the attached wave (associated) to a particle for any physical size that characterizes the particle (momentum, position, spin, etc.). These two parts propagate in space at a different speed, so that the particle moves at finite speed, while the attached wave propagates immediately at infinite speed². This phenomenon is called the "non-locality" of the wave function - the wave function is not local, it has no position from which it propagates at a limited speed (for example, the speed of light), but the propagation is immediate throughout space. It is important to emphasize that when we say that a particle is simultaneously in several different states (classically contradictory to each other), so that each state has a different probability, we do not mean that the state of the particle is unknown, and there are different probabilities of finding out its true state. The exact state of the particle is known to us absolutely, and it consists of different states in different probabilities. In such a case we say that the state of the particle consists of a superposition (in Hebrew: composition) of different states.

What happens at the moment of measurement? Once we have measured the physical magnitude and obtained a certain value, which is one of the multitude of values that made up the situation, henceforth this value is the only one that exists. We describe this with the words "the wave function collapses and becomes an eigen(self)-state with the same specific value" found in the measurement. If we now make additional measurements, we will all get the same value we got in the first measurement. All other values that were possible, and together built the wave function, disappeared when the state "collapsed" to the particular value observed in the first measurement.

Raspberry Activity: A comparison between classical physics and quantum physics



Some readers may think: It's simple. There was always a certain value, but until the measurement we did not know what that value was, so we talked about a lot of possible values. The measurement did not change anything, but only told us what the same value is, and indeed this value that was the only one all the time, will continue to be the only true value even now. However, this is not a correct description of the physical reality, as we will prove later with the help of experiments performed. Before the measurement, not only did we not know what the true value was, but there really was no single true value. The state of the particle was a collection of different states, it was the true state of the particle, and now its state has changed and become a single-valued state.

Raspberry activity: Understanding the material waves - video-based activity. Students watch two short videos and answer questions about the material waves.



²Einstein opposed this view, since it contradicts the principle that nothing can pass faster than the speed of light. At the same time, we now understand that indeed the wave function spreads throughout space immediately without limitation of the speed of light. Still, there is no material contradiction to the theory of relativity since information cannot be transmitted at a speed greater than the speed of light by this.

Summary so far

In contrast to classical physics in which an electron and the like is a particle, in quantum physics the particle is also a wave. The electron has particle properties and wavy properties.

In contrast to classical physics in which a wave of matter represents the amount of matter, here the wave (or wave function) represents the probabilities of obtaining different results in measurement.

In contrast to classical physics in which measurement gives us information that was not known to us but existed, in quantum physics measurement actually determines the state, and changes it from a state of superposition, to a single state.

In the epistemological context we mentioned in the opening, the features we mentioned here regarding classical physics belong to the periphery, whereas in quantum physics - this is the nucleus. In the body are the applications and examples.

As an important example it is recommended to watch the video "Dr. Quantum And the experiment of the two slits" On interference with waves and electrons. It is worth stopping during the film and discussing with the students different points.

<https://www.youtube.com/watch?v=yS3DXQ9jY04>



Raspberry Activity: Watch the video "Dr. Quantum and the Experiment of the Two Slits" And answering questions during the video.

In the quantum case there is a collapse of the wave function at the moment of measurement. The measurement can take place in two places:

A. On the screen, then the electron hits one possible position, and when shooting many electrons see an interference pattern.

B. If the women put a measuring instrument in one of the cracks. In this case, we do not see an interference pattern.

Many students have already encountered the video before learning the subject, but it is usually difficult for them at first to connect the phenomena in the video with what we have just learned. It is worth letting them predict what will happen at each stage, and explain this in light of the basic principles of quantum physics.

When we put a detector in one of the cracks the wave function collapsed so we did not see any interference. This illustrates what we have said that the measurement changes the wave function and causes the superposition to collapse, rather than just providing information about the situation. If it provided information only, there should have been no difference between the presence of the detector in the crack and its absence.

Let us pay attention here to the phenomenon of non-locality. While the electron is moving at some finite speed, its waves, or its wave function propagates in space immediately without limitation of velocity. This is how the electron "knows" if there is a detector in the second slot,

and this is how interference occurs even though the different orbiting paths are not of the same length.

The quantum state

A physical state is actually a description of the value of a particular physical quantity or of a number of physical quantities. In classical physics, the position of a particle and its velocity (or alternatively its position and momentum) are often discussed in order to describe its state.³ The uniqueness of the quantum state is that it can consist of a superposition of different “classical-like” states.

A quantum state with a single value of some physical magnitude is called a eigen-state of the same magnitude, as opposed to a non-eigen-state called a superposition state.

Worth emphasizing: A particle or body cannot be at a given moment in two different places (or with two different energies or two different motives). This is not possible in either classical or quantum physics. The state of superposition does not express being in two places (or more), but probabilities of being in two places. Again we will not mean that we do not know where and there is one chance or another that the particle is in a certain place, but the state itself is a state of superposition containing different states in different probabilities. Supposedly, nature itself does not know.

In classical physics there are situations, especially in multi-particle systems such as a container full of lots of gas molecules, in which we describe certain sizes statistically, since we do not know the velocity of each particle and particle. In quantum physics (meanwhile we are talking only about one particle), when we talk about probabilities we do not mean lack of knowledge, but about knowing a state that is not of single value but a state that is a superposition of different states in different probabilities.

The misconception that because of the probabilistic nature the result of the experiment is not accurate is also worth emphasizing: the probabilistic nature does not harm the accuracy of the experiment - the distribution of the probability can be predicted with very high accuracy. Accuracy is in determining the probability of getting a certain value in a certain measurement, and this probability quantum physics knows how to calculate with very high accuracy and in very good agreement with the measurements.

It is worth emphasizing to students these differences on all sorts of occasions, in order to reduce the chance of misunderstanding.

³ Later it seems that in quantum physics it is not possible to talk about position and momentum at the same time, but at the moment it is not relevant.

Heisenberg's uncertainty principle



ורנר הייזנברג, 1901-1976

In 1927, the German physicist Werner Heisenberg discovered the uncertainty principle. The principle states that we can never know with absolute accuracy at the same time the position of a particle and its momentum. The wording of the law is⁴:

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2}$$

Here we express our error or inaccuracy in knowing the position or momentum $\Delta x, \Delta p$. The more accurately we know one of these two sizes, the greater our minimal error in knowing the other size. It should be emphasized that this is not a lack of knowledge or a technical limitation related to the production capacity of good measuring instruments. This is a fundamental inability related to the nature of states in quantum physics.

A popular explanation for the uncertainty principle is called the "Heisenberg microscope" and it explains the principle as follows: in order to know the position of the particle we need to see it, for example by a photon. However, when the photon hits the particle it will give it momentum, so we can not know what its momentum is. The more accurately we want to know the position, we will need a photon with a smaller wavelength (wavelength is actually the scale of accuracy in position), but then the photon will have a greater momentum, and our uncertainty about the momentum of the particle will be greater anyway. This explanation is problematic and therefore it is recommended not to bring it to the students. It reinforces the understanding of the uncertainty principle as a technical difficulty in knowing the position and momentum at the same time. It links the uncertainty to our knowing, rather than to something essential in the properties of the particle. This explanation is a misconception, and is part of the periphery of quantum physics.

In fact, some argue that the name "uncertainty principle" is a distorted translation from German, and it is a misleading concept, which gives the wrong feeling as if only for us it is not certain, while it is a fundamental inability to define such accuracy. A concept that can replace "uncertainty" is indeterminacy - that is, the inability to determine.

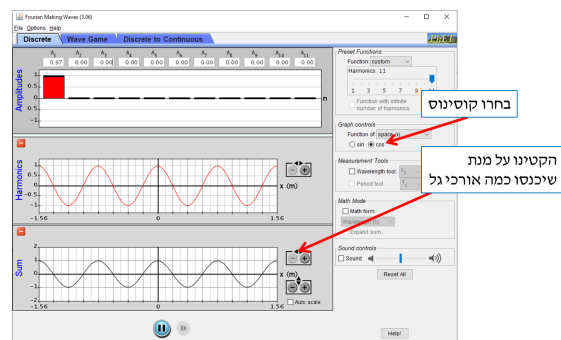
Another wording of the law says that we can never know with absolute accuracy the momentum and position, i.e. there can be no state that is both an eigen-state of position and an eigen-state of momentum. If a state is a eigen-state of position, it is certainly a state of superposition of many momentum values, and vice versa, if it is a state with a definite momentum - eigen-state of momentum, then it must consist of different position states. There can of course be a state that is not an eigen-state neither of the momentum nor of the position.

We can describe a state as a superposition of position states. In this case it is said that we describe according to the basis of location. Alternatively it can be described by the momentum of momentum. Eigen-states of momentum will always be described as a superposition when the description is in the base of the position, and vice versa. We will return to this description later.

⁴ We used here the reduced Planck constant: called: $\hbar = \frac{h}{2\pi}$ called h-bar.

Demonstration of the uncertainty principle through simulation

We would now like to illustrate the principle of uncertainty.



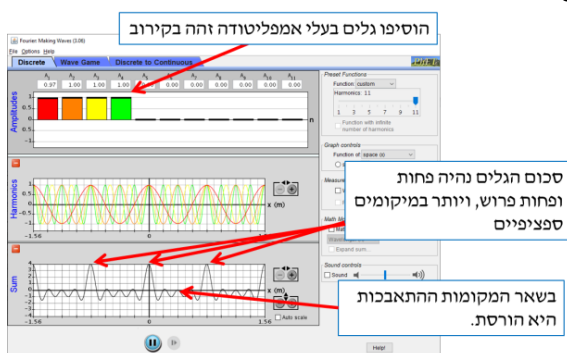
It will be recalled that according to de Broglie waves there is a relationship between momentum and wavelength. When we mean wavelength it means the length of a period of sine wave or cosine function. If the periodicity is not sinusoidal, including infinite (i.e., there is no periodicity), we can break it down into sines and cosines by means of Fourier decomposition. An eigen-state of momentum is actually a state with a single momentum, and in any case a state with a single wavelength - sine or cosine or a certain combination of both. To be more precise, a sine-only or cosine-only wave is a super-position wave of two momentum identical in size and inverted in their direction e^{ikx} (i.e., still of the same size, and thus, in no way contradictory to de-Broglie waves). A single momentum wave is one of a kind, but we do not want to deal with complex numbers here at all. Therefore we will talk about a sine wave or cosine and it is said that it has a single size of momentum, as it appears in the de Broglie formula. It is clear that in this case a particular location has no meaning,

but the wave is in any location.⁵

A defined location seems to require an infinite multiplicity of motions (wavelengths). It is recommended to use the class in the simulation at the link: <https://phet.colorado.edu/en/simulation/fourier>

The simulation shows us a superposition of several waves. In the upper rectangle the amplitude of waves of different wavelengths can be determined. In the middle rectangle all the waves are depicted in different colors. The lower rectangle depicts the interference of all waves. In the lower rectangle, the axis X is a position axis. If the interference of the waves is of high value, it means that the probability of finding a particle in this place is high.

For the purpose of demonstration, it is advisable to work with cosine waves. We want to show that in order to create a defined



⁵ Again, there is a difference here between the wave that is commonly spoken of, which has a constant probability in space, and a wave of sine or cosine that has reset points. e^{ikx}

location we will need an infinity cosine wave superposed. You should lower the graph (at the bottom) so that we can see the number of wavelengths on the interference screen. In the first stage, one wave appears, with the largest wavelength.

In the second stage, in the upper rectangle an amplitude (more or less uniform, or gradually smaller) should be given to each of the waves. With each addition, we see that the interference between the waves resets almost the entire area between the large peaks. It can be noted to

students who have learned this, that it is a phenomenon that is somewhat reminiscent of diffraction from lattice, where multiple peaks appear with destructive interference between them.

The distance from one peak to another is equal to the length of the largest wave.

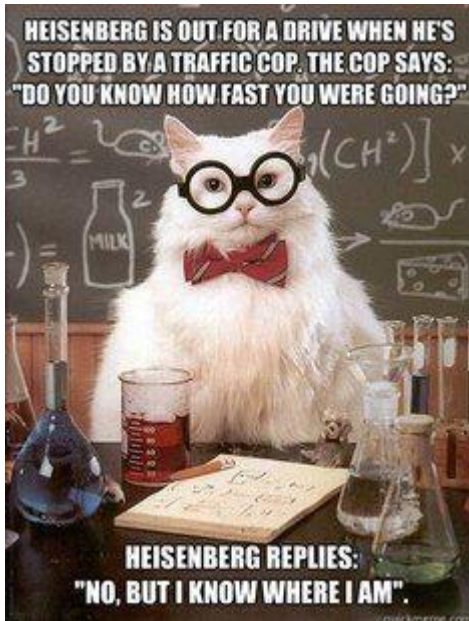
Remember that each such wave represents a certain momentum. The smaller the wavelengths (large momentum!), The more concentrated the peaks will be:

The conclusion from this demonstration is that we need an infinite number of wave in order for the combined wave function to be constructed of very sharp peaks when the wave function is zero between them.

However, there is still no definite location here, but a set of an infinity points, at fixed distances between them, where only the particle can be found. How can we reach a single, defined location?

In this simulation, we started with using a certain wavelength, and added smaller and smaller wavelengths. The distance we obtained between the points where there is a high probability of finding the particle was the first wavelength (large, small-momentum). In the simulation we are limited, but if we were to start the process from a larger wavelength (small momentum!) The distance between the resulting peaks would be greater. In order to reach one and only one very concentrated peak, around which everything (almost)⁶ goes to zero, all wavelengths are needed, from very large (small momentum) to very small (large momentum). This is in fact an illustration of the principle of uncertainty, which shows that in order to be in a definite place, the superposition of many waves is needed.

⁶If there are indeed infinite wavelengths, then the wave function will be zero between peak and peak. If there are many wavelengths but in a finite number, then it will be nearly zero, but in practice we can treat it as zero.



The reason why the explanation "Heisenberg's microscope" described above is quite common (for example, in Hawking's book⁷), is because it provides an intuitive explanation for understanding the principle of uncertainty. This is an epistemological explanation, an explanation that says we have a "technical" problem that we cannot know the two sizes simultaneously. The explanation presented through the simulation is an ontological explanation, that is, refers to the physical reality, how nature actually behaves. This explanation makes it clear that the uncertainty principle stems from the material waves: since the particle has wavy properties, and the wave property is immanent to physical quantities like momentum and position, to get a certain position it takes interference of lots of momenta and to get a certain momentum it needs interference of many positions. In fact, as we have said, there are other pairs of physical quantities

between which there is uncertainty ("non-commuting operators"), and this is not necessarily related to location and momentum. For other quantities the explanations given, both the Heisenberg microscope, and the description we presented using the Fourier series, are irrelevant, but as stated, this uncertainty principle can be proved mathematically for all such pairs of quantities.

Applications of the uncertainty principle

We will present here two examples of applications of the uncertainty principle. As part of the division into the nucleus, body and periphery of the knowledge of quantum physics, these applications belong to the body.

1. The absolute zero of motion in a liquid/solid cannot be reached when cooling a system. When a material cools, the material shrinks and the motion of the particles slows down. At first this is not a problem, but as one approaches absolute zero, the closer the particles are, the greater the certainty of their location since a particle is blocked by its neighbors, and in any case the momentum uncertainty increases, i.e. the velocity of the particles increases. Absolute zero motion in a solid is a state in which all the particles stand in place, and this is a state of a definite position and a definite momentum, and therefore this is impossible.
2. Harmonic Oscillator - In classical physics there are oscillations with angular frequency ω , and any energy can be set in in the system. In quantum physics the possible energy levels are, $E_n = \hbar\omega \cdot \left(n + \frac{1}{2}\right)$ when n is a non-negative integer. We see in this expression, "quantization of energy level values": that is, the permissible energy does not receive any value, but depends on n , which is an integer, and therefore defines discrete values. Beyond the quantization of energy level values, it can be seen that there can be no zero energy, but the minimum is some kind of energy. Admittedly, very very small in everyday terms. This is due to the principle of uncertainty: if there was zero energy - then the body


⁷ Stephen Hawking, "A Brief History of Time," p.

would stand in a certain place. "Standing" means having a definite momentum (zero), in a certain place, that is, in a definite place at the bottom of the potential well, and this is contrary to the principle of uncertainty.

The uncertainty principle can be extended to other physical quantities as well. There are pairs of physical sizes that can never have a common eigen-state. The eigen-state of the one will require the superposition of the eigen-states of the other. There is a relatively simple mathematical way (but goes beyond our field of study⁸), Through which such pairs of physical sizes can be identified.

The principle of uncertainty is part of the fundamental principles of quantum physics, and is therefore at its core. True, it is mathematically derived from more basic principles, but as part of our study in high school, this cannot be explained, so we will present it to students as part of the nucleus. This is an essential innovation that exists in quantum physics, and has no equivalent in classical physics. We will emphasize again that this is not a classical uncertainty resulting from a lack of knowledge, but an essential principle in quantum physics.

As an example it is recommended to give students the worksheet on measuring and collapsing the wave function in a two-level system presented below and comparing the theoretical understanding and the results of the experiment using simulation (in the following pages).

 *Raspberry activity: Measurement and collapse of the wave function in a two-level system. This activity is intended to connect the chapter that dealt with the measurement and waves of matter and the principle of uncertainty. In this activity the student becomes acquainted with the concept of spin, and the uncertainty that exists between its components.*

The following pages provide an example of a raspberry-like worksheet. In the example the students start from predicting results based on their theoretical understanding and can proceed to an experiment that can be conducted through simulation. The experiment in the simulation illustrates what they saw theoretically. The experiment demonstrates how the wave function collapses, and the electron "forgets" the results of previous measurements, but if the same measurement is repeated twice in a row (without measuring in the vertical direction in the meantime), the measurement result will be preserved. In this experiment it is difficult to see what the result of measuring in a direction other than one of the axes will be x or z. To see the angular dependence (the cosine square of the angle) it is recommended to perform a quantitative experiment in which the angle is changed and the percentage of electrons passing is recorded. We brought the results of the experiment after the worksheet.

Worksheet: Measuring and collapsing the wave function in a two-mode system

We explained that the wavelength of a particle means that the particle can be in a state that is a superposition of different locations, at different probabilities. We emphasized that this does not mean that we do not know the location of the particle, but that its location is not defined as one

⁸This is related to the alternation of the operators representing the physical quantities. If the operators are alternate they can find common self-states, if they are not alternate they have no common self-states.

location but as many locations, and this means a probability that will be expressed in the case of measurement.

We have expanded this definition not only for location, but for any physical quantity such as momentum, energy, angular momentum and more. A particle can be in a state that is a superposition of different states, even if classically a physical property of a particle is uniquely determined, and it is not possible for a particular particle to have two values of the same physical size.

We have seen that the uncertainty principle sometimes does not allow different physical quantities (such as position and momentum) to have a single value at the same time, so if in terms of one quantity we are in a state where size has a defined single value, the other physical quantity must be superimposed on different states. This means that if we measure the value of the one magnitude, the wave function will collapse anyway and we will get the particle in a single defined state of the measured magnitude. In this situation it will be assumed that the second magnitude will certainly be in a state of superposition of different states, even if before the measurement it was in a single defined state.

In this worksheet we would like to talk about a new physical size that you have not yet learned about - the spin (Hebrew: ספִּין). What is a spin? The most convenient way to observe the spin is to treat it as a rotation of a particle around its axis. When we say "an electron has a spin" it means that we can construct a model in which the physical-mathematical description is such as if

the electron has a spin. In fact, there is really no spin as there is really no electron - there is a model that describes the world. The electron is an idea. A way to represent the world. So far, this way, is consistent with the results of experiments, so it is a successful model. Therefore, we can say: "There is an electron, and an electron has a spin."

According to our model, the electron, as a particle with an electric charge, produces a magnetic field in its motion. In fact, the magnets we encounter are usually formed by the fact that the spins of the particles are all arranged in the same direction, and thus they connect to the size of a significant magnetic field. Also the medical imaging device MRI is based on measuring the spins of the protons in the body.



In an experiment conducted in 1922 by German physicists Otto Stern and Walter Gerlach, it became clear that the electron had a spin that could have only two values:.. We will not focus on this in the current framework, and will simply refer only to the sign of the spin value, noting that each measurement of the spin can have one of two results: a positive spin or a negative spin. The spin of the electron can be measured in any possible direction, and we will always get in the direction we tested one of the two results: a positive spin or a negative spin. In Stern's experiment and your measurements the measurement was made by moving the electron through a changing magnetic field. The direction of change of the magnetic field is the direction in which we measure the spin. If we take an electron at random and measure its spin in a certain direction there is a 50% chance that we will get a positive spin, and a 50% chance that we will get a negative

$$\text{spin. } \frac{\hbar}{2}, -\frac{\hbar}{2}$$

It turns out that similar to position and momentum, according to the principle of uncertainty, the spin in its various directions is not known at the same time. In other words, a spin is defined in a certain direction, which we mark as an axis, is necessarily a state of superposition of a spin in a vertical axis perpendicular to it (y or z). That is, if we measure the spin in the direction of the x-axis we get a certain result. In such a case the wave function that existed would collapse, and now the electron will have a single and defined spin-x (and not a superposition) and necessarily a state of superposition (with a probability of 0.5-0.5) in its vertical axis (y or z). In the same way, a spin defined on the z-axis is necessarily a state of superposition (with a probability of 0.5-0.5) on the x-axis.

Because spin has only two possible modes in each measurement, a spin measurement experiment can be a convenient example of measuring and collapsing the wave function, due to the simplicity of the probability calculations in a small number of different possibilities.

We will now describe a number of possible experiments. In each of them we indicated what the result of the experiment would be. If there is more than one result, indicate the results and what will be the chance (in exact percentages, or in a rough estimate) of getting each of the results. As mentioned, in any measurement the result can be + or -.

Next we will check the results using computer simulation. At this stage the results should be recorded according to your theoretical understanding only.

- 1. Take a random electron and measure its spin in the axis direction x.
outcome:*

- 2. Measure the spin in the direction of the axis to the electron x and the result + was obtained. Now measure his spin again in the direction of the x-axis.
outcome:*

- 3. Measure the spin in the direction of the axis to the electron x and the result + was obtained. Now measure its spin in the direction of the z-axis.
outcome:*

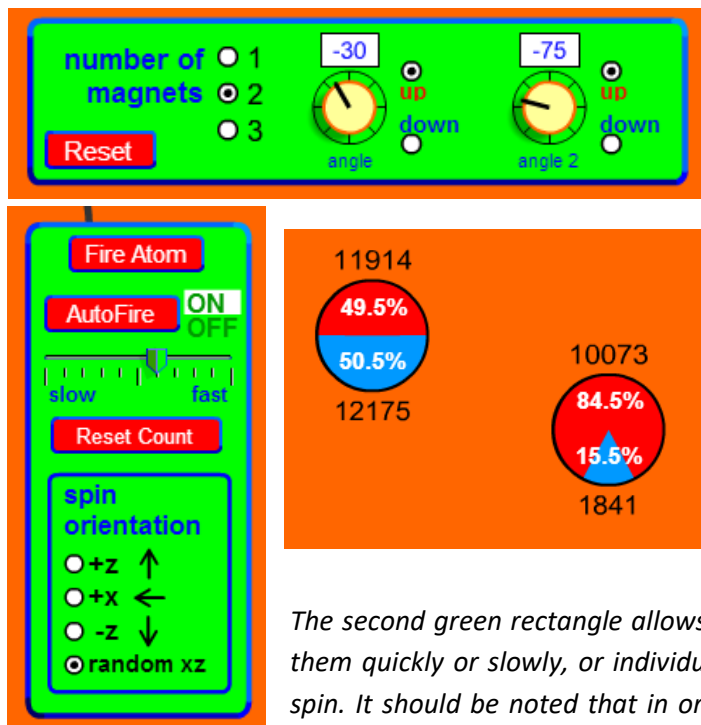
- 4. Measure the spin in the direction of the axis to the electron x and the result is obtained. Immediately afterwards his spin was measured in the direction of the z-axis and the result + was obtained. Now measure his spin again in the direction of the x-axis.
outcome:*

5. Measure the spin in the direction of the axis to the electron x and the result is obtained. Immediately afterwards measured his spin in the direction of the z-axis and the result was obtained -. Now measure his spin again in the direction of the x-axis. outcome:
6. Measure the spin in the direction of the axis to the electron z and the result + was obtained. Now measure its spin in the direction of the z axis.15° outcome:

We will now try to test these experiments with the help of computer simulation. We will go to the following link:

https://phet.colorado.edu/sims/stern-gerlach/stern-gerlach_en.html

In this simulation we will be able to perform the experiments we tried to answer. There are two green rectangles that control the experimental system. In the top rectangle we can determine the



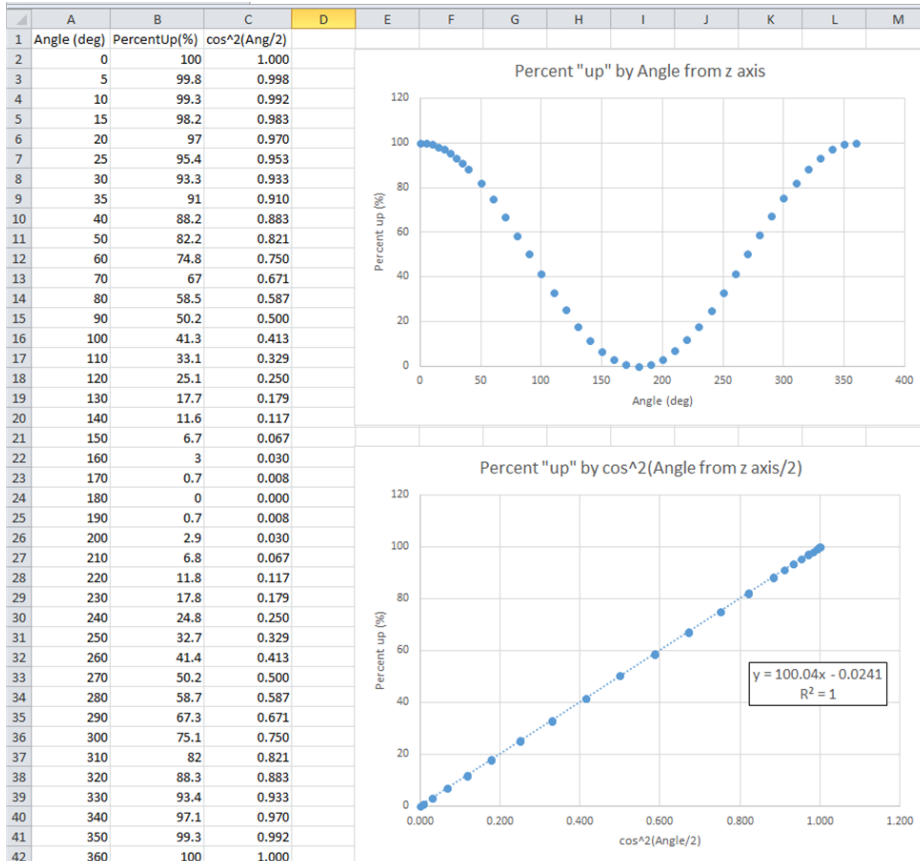
number of spin measurements made one after the other (each measurement by a magnet, up to 3 measurements - as was the case with the questions we asked earlier). You can also determine the direction of each measurement. Note that the angle indicates the axis 0° z while the angle indicates the x-axis. In each detector we can choose which electrons can proceed to the next detector - for those that received a positive spin value (up) or for those that received a negative spin value (down). Other electrons will be blocked and will not move to the next detector. In this way we can, in case we know the result in one measurement, know what will happen in the measurement that follows. -90°

The second green rectangle allows us to control the measured electrons. You can shoot them quickly or slowly, or individually. You can give them a pre-determined or random spin. It should be noted that in order to arrive at a result that correctly represents the statistical probability, measurements must be made on many electrons.

The measurement results for each detector are presented both in detail of the number of electrons that received a positive spin value and the number of electrons that received a negative spin value, and as a pie chart indicating the positive percentage (red) and the negative percentage (blue).

We will try to examine the results for all the experiments we presented earlier, and compare with the results we observed. In some cases we will try this in a number of ways. We will try to understand why the different ways are in fact equivalent ways to the same measurement.

1. *Select a random spin in the left rectangle, and measure the spin x (i.e., in the top rectangle an angle of) is selected. We will launch many electrons and see the statistics.— 90°
outcome:
We will repeat the same measurement with a spin set in the direction of the axis z . Will we get the same result? why?*
2. *Launched electrons with spin x Positive. We will measure spin $-x$.
outcome:
Launched many electrons with a random spin, measured spin x , and we will only allow the results up to continue. For the electrons that continue, we will measure the spin again in the direction of the $-x$ axis. Will we get the same result as in the case of launching electrons with a positive spin x ? why?*
3. *We will take random electrons and measure their spin x and then the spin z . A parallel experiment is to measure spin z for electrons with a positively defined spin x .
outcome:*
4. *Random electron launches, we measure the spin in the axis x , let the results down continue, we will measure the spin in the direction of the z axis, let the results up continue, and we will measure again in the x axis.
outcome:*
5. *Random electron launches, we measure the spin in the axis x , the down results can be continued, we will measure the spin in the direction of the z axis, the down results can be continued, and we will measure again in the x axis.
outcome:*
6. *Random electron launches, measured in direction z and allow the positives to continue (in a parallel experiment it is possible to start with electrons with a positive spin z and give up this measurement). Now measured at an angle of 15°
outcome:*



The image on the left shows the graph obtained from inserting the simulation results into a computerized datasheet. Change the angle of the magnet and record the percentage of electrons with the spin "up".



Raspberry activity: an experiment in which the percentage of electrons with spin is measured "up" detected by the detector depending on the angle of the magnet.

התלות של אחוז האלקטרונים עם ספין "up" בזווית המדידה – תוצאות סימולציה

Chapter 3 - The Mathematics of Quantum Physics

As stated, the wave function of a particle describes the probability that the particle will be in one position or another. Sometimes the wave function is not about the probabilities of different locations but about the probabilities of being in some physical state.

We will now want to build the mathematical system with which we will work in quantum physics.

Mark Dirac

We will mark a physical state as follows: Inside the bracket you can write any number, letter or sign that will define the situation: $|\psi\rangle$, $|a\rangle$, $|b\rangle$, $|1\rangle$, $|2\rangle$, $|\uparrow\rangle$, $|\downarrow\rangle$...

Usually we are talking about the set of self-values that are relevant to the situation and not about the physical sizes that are in the superposition.

It should be understood that different self-states, that is, different values of the same physical magnitude are orthogonal - that is, contradictory (classically).

You can also mark the situation as follows: There is no difference between the meanings of the markings $\langle\psi|$ ⁹, But in most cases we will mark in the first way we have presented, as detailed below.

Suppose we have two states of some kind, which describe certain physical quantities, (read: "Pi", "Pesi"). The situations are not necessarily contradictory, so given that we are in a situation there is some probability that in a proper measurement, we are found to be in a situation. We will define this probability as follows: $|\psi\rangle, |\phi\rangle|\psi\rangle|\phi\rangle$

$$\langle\phi|\psi\rangle^2$$

We emphasize that this is true if composite numbers are not included. If you choose to include complex numbers (and this is the more comprehensive presentation) the absolute value of the expression must be squared. $\langle\phi|\psi\rangle$

In words: the probability of getting the situation measured, given that it is known (from a previous measurement) that we are in the situation. $\phi\psi$

Note: At this stage, it is not yet clear to the students what the arithmetic exercise needs to be done to get to the stage where something needs to be raised squared, rather than what is raised. This is a definition only.

The check $\langle\phi|\psi\rangle$ Expresses the overlap between the modes. These are parentheses (bracket), so we denote a condition by marking the machine ket or by marking the machine bra. There is no physical difference between bra and ket. $|\psi\rangle\langle\psi|$

Probability of gaining self-worth in the case of a superpositional state of two contradictory states

To understand the meaning of this we will first talk about two contradictory states, i.e. two states that each describe a single value (not a superposition) and a different one of a certain physical magnitude. We will mark the situations. We have assumed that the situations are contradictory and therefore, if we are in the situation $|b\rangle, |a\rangle$, the chance of finding the value a in the measurement is zero. We will mark this: . In words: the chance of measuring a given that we are in state b is zero. Therefore we know that for conflicting situations, their overlap is zero: . In addition, it is clear that if we are in state a, then in the measurement we will get the value a. Therefore, the chance of getting a is 1. We will mark this: . Mathematical reasons not listed here are accepted (and not $1-\langle a|b\rangle^2 = 0\langle a|b\rangle = 0\langle a|a\rangle^2 = 1\langle a|a\rangle = 1^{10}$).

Important fact (we will not prove its correctness here): it should be emphasized that given two situations (not necessarily contradictory) $|\psi\rangle, |\phi\rangle$, Then $\langle\phi|\psi\rangle = \langle\psi|\phi\rangle$. That is, if when we are in

⁹ In fact, the difference between the notations is in terms of the mathematical representation, and less in terms of the physical meaning, so in the context of the material studied here we will not address the difference between them.

¹⁰ We here completely ignore the mathematical issue of complex numbers. The positive value is obtained because of the positive property of the inner product. The product of each vector itself is a non-negative quantity.

a state there is a certain probability of being in a state ψ (E.g., a probability of 0.3), then if we are in a situation, we will have the same certain probability (0.3) of being in a situation ψ

We will now look at a state that is a superposition of two conflicting states **a** and **b**. We will record this as follows:

$$|\psi\rangle = A|a\rangle + B|b\rangle$$

When they are some numerical coefficients (in fact they can not get any value, we will see this later). Remember, as soon as we make a measurement, the wave function will collapse, and we will have one and only value - self-value, of physical size. We can get in measurement or A, B a or b. What is the probability that given the state of the superposition we will get in measurement the value a (i.e., that we are in the state)? $\langle a|\psi\rangle$

We will perform the mathematical move of the overlap:

$$\langle a|\psi\rangle = \langle a| \cdot (A|a\rangle + B|b\rangle) =$$

Students' attention should be drawn to the procedure: place the appropriate place in the overlap. Then perform multiplication: $\langle a|\psi\rangle$

$$= \langle a|A|a\rangle + \langle a|B|b\rangle = A\langle a|a\rangle + B\langle a|b\rangle =$$

The constants are allowed (and should) be taken out of bracket. Remember, we do not justify the procedure mathematically, just explain how to do it.

We have seen above, that the physical meaning of is to find size in measurement $\langle a|a\rangle$ Given that we are in state a (remember, state a is not a state of superposition), and the probability of this is 1. Also, we have seen that the physical meaning of it is to find the magnitude a given given that we are in state b (also b is not a state of superposition), and because they are contradictory states, The probability of this is 0. Therefore: $\langle a|b\rangle$

$$= A \cdot 1 + B \cdot 0 = A$$

And so the probability is. Similarly the probability of being in a situation is $\langle a|\psi\rangle^2 = A^2|b\rangle B^2$ ¹¹. That is, in a state that is a superposition of different states (even in the case of more than two states), the coefficient of the square of each state and state expresses the probability of being in the same state. It follows that if and are the only possible values, in this case it should exist. The coefficients can also be negative, but never will any coefficient be less than or equal to its absolute value of 1. $|b\rangle|a\rangle A^2 + B^2 = 1$

Probability of getting self-value in case of superposition mode of two superposition modes

We will now want to learn how to calculate overlaps and probabilities between two superposition modes. Suppose a system has two possible conflicting states, for example: two possible locations. Let us look at the two superposition modes of the position. The first state is a state with some definite energy, that is, a self-state of energy. This mode is a superposition mode of the placements. We will express it as follows: $|b\rangle, |a\rangle\psi E_1$

$$|\psi\rangle = A|a\rangle + B|b\rangle$$

¹¹ It is worth noting to students that in general it is possible to talk about composite coefficients, and then the probabilities are the square of the absolute value of the coefficients.

The second state, the state, which is not a self-state of energy, and is not a self-state of position, but a self-state of momentum, that is, a state with some definite momentum that we signify. Since this state is not a state of self of the position, but a state of superposition of the positions, we will express it as follows: φp_2

$$|\varphi\rangle = C|a\rangle + D|b\rangle$$

Of course, are numerical coefficients that meet the necessary conditions, that is, the equations are met as well. $A, B, C, D, A^2 + B^2 = 1, C^2 + D^2 = 1$

Note that if the two states are not contradictory they are not necessarily self-states of the same physical magnitude. After all, different self-states of one physical magnitude are necessarily contradictory - it is not possible for us to be in two different positions (or energies, or motives, etc.) at the same time. To do this, we chose two modes, each of which is a self-state of a different magnitude - of energy and of momentum. At the same time, in this calculation we have expressed each of these states as a superposition of position states. For simplicity we have chosen here in a world where there are only two location modes. We will mention again that when we describe any situation according to the self-states of the position we say that "we describe according to the basis of the position". According to the principle of uncertainty, when describing the self-state of momentum with the aid of the position base, it will never be described as a superposition of different position states. $\psi, \varphi\psi\varphi$

We are interested in knowing, given that we are in a situation, what the probability is of being in a situation. We will emphasize the physical meaning of the question: Given that we are in a state, that is, we measured the energy and the result obtained in the measurement was, that is, we are in a state; What is the probability of being in a state, that is, if we now measure the momentum, what is the probability that the result obtained is $\psi\varphi\psi E_1\psi\varphi p_2$

We will now perform the calculation. For the purpose of the calculation we need to calculate the overlap, i.e. the overlap between the state of the $\langle\varphi|\psi\rangle$ good and ket mode. To do this, we will need to write the situation as a good situation. This is done simply by turning each ket into a bra, i.e. $\therefore\langle\varphi||\psi\rangle|\varphi\rangle$

$$\langle\varphi| = C\langle a| + D\langle b|$$

And the overlap will be:

$$\begin{aligned}\langle\varphi|\psi\rangle &= (C\langle a| + D\langle b|)(A|a\rangle + B|b\rangle) = CA\langle a|a\rangle + CB\langle a|b\rangle + DA\langle b|a\rangle + DB\langle b|b\rangle = \\ &= CA \cdot 1 + CB \cdot 0 + DA \cdot 0 + DB \cdot 1 = CA + DB\end{aligned}$$

Thus, the probability of measuring momentum given the energy state is: $p_2 E_1$

$$\langle\varphi|\psi\rangle^2 = (CA + DB)^2$$

Of course, if the coefficients are not real, then the absolute value of the overlap should be squared.

Another example:

Note that both the previous example and the example below are examples related to the physical body of knowledge of quantum physics. They are not the nucleus, because Dirac marks and the mathematics that accompanies them are only a "means" to be able to create predictions for experiments. Dirac markings are not "deep physical principles", but a way of expressing the principles using a page, pen and calculator.

Take for example the spin modes. There is a base of spin (reminder: "base" means the physical size by which we represent the state). His limbs are marked (spin left; spin right), and there is a base of spin, his limbs are marked (spin up; spin down). When talking about the spin of the particle in the

axis $x| \rightarrow \rangle, | \leftarrow \rangle_z | \uparrow \rangle, | \downarrow \rangle_z$, then in the x -axis it is in the superposition. Therefore, the spin organs can be presented as follows:

$$| \uparrow \rangle = \frac{1}{\sqrt{2}} | \rightarrow \rangle + \frac{1}{\sqrt{2}} | \leftarrow \rangle$$

$$| \downarrow \rangle = \frac{1}{\sqrt{2}} | \rightarrow \rangle - \frac{1}{\sqrt{2}} | \leftarrow \rangle$$

- Note that the selection of the coefficients' marks is almost arbitrary: what is important is that in one of the "up" or "down" modes the coefficients will have a mark Same (in our example, in "Spin Up"), And in the second situation have a mark Reverse (one positive coefficient and one negative). This is to ensure that the situations are contradictory, that is, their overlap will give zero, since in the measurement we will not be able to get the two values "spin up" and "spin down" together (think and make sure that the overlap is reset!). It is worth reminding students that the equation should match the result of the measurements and not the other way around ...
- The numerical values of the coefficients () are due to the fact that there are only two possible spin modes and they have an equal probability. Remember, the coefficient square is the probability of getting the measurement value. $\frac{1}{\sqrt{2}}$
- Students should be given an exercise:
Calculate the overlaps. Remember the results of the activity we did in the Stern-Gerlach experiment, as the results match what we saw in that activity: $\langle \uparrow | \uparrow \rangle$, $\langle \rightarrow | \uparrow \rangle$, $\langle \leftarrow | \uparrow \rangle$, $\langle \uparrow | \downarrow \rangle$, $\langle \rightarrow | \downarrow \rangle$, $\langle \leftarrow | \downarrow \rangle$

Example: What is the probability that a spin will be measured "left" (vintage) x) Given that we measured a spin "down" (in the z -axis)?

$$\langle \leftarrow | \downarrow \rangle = \langle \leftarrow | \cdot \left(\frac{1}{\sqrt{2}} | \rightarrow \rangle - \frac{1}{\sqrt{2}} | \leftarrow \rangle \right) = \frac{1}{\sqrt{2}} \cdot (\langle \leftarrow | \rightarrow \rangle - \langle \leftarrow | \leftarrow \rangle) = -\frac{1}{\sqrt{2}}$$

So the probability is as we know, that the chance is 50% to get a spin left given that the situation is down. $\left(-\frac{1}{\sqrt{2}} \right)^2 = 0.5$

Note: When calculating such overlaps with the students, then in the first stages write the states of the bra and ket modes, open parentheses and calculate all overlaps. In more advanced stages this can be done more quickly, and show students that the only things that contribute to the overlap are the multiplications of the coefficients of the same organs. For example, if the data states:

$$| \psi_1 \rangle = a | 1 \rangle + b | 2 \rangle + d | 3 \rangle$$

$$| \psi_2 \rangle = e | 1 \rangle + f | 2 \rangle + g | 3 \rangle$$

After all, the overlap will be:

$$\langle \psi_1 | \psi_2 \rangle = ae \langle 1 | 1 \rangle + bf \langle 2 | 2 \rangle + dg \langle 3 | 3 \rangle = ae + bf + dg$$

Here we have omitted all the reset overlaps.

Another example:

We now want to treat measurements in different directions, and not just in directions that are perpendicular to each other.

We have seen in the past, with the help of imaging, that given a spin in a certain direction, then if we measure the spin at an angle θ From the original direction, the chance that we will get a positive spin name is given by the expression $\cos^2 \frac{\theta}{2}$. Hence of course it follows that the probability that there is a negative spin, i.e. opposite in its direction, is the complementary probability, i.e. .:

$$1 - \cos^2 \frac{\theta}{2} = \sin^2 \frac{\theta}{2}$$

From this the situations can be written with the help of Dirac markings as follows:

$$|\theta^+\rangle = \cos \frac{\theta}{2} |x^+\rangle + \sin \frac{\theta}{2} |x^-\rangle$$

Meaning: The condition means a positive spin in the measured direction, i.e., $|\theta^+\rangle$ At an angle θ From the axis, which was the original direction, and consists of a superposition of the various spin states in the original measuring base (which was in the direction or in the opposite direction to it, it is sometimes convenient to mark the whole base $x^+ x^-$ in brief: x^\pm

$$|\theta^-\rangle = -\sin \frac{\theta}{2} |x^+\rangle + \cos \frac{\theta}{2} |x^-\rangle$$

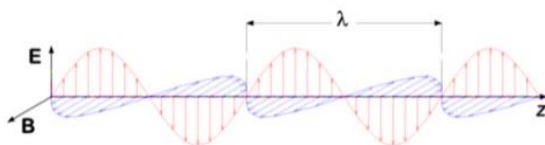
Meaning: The situation means a negative spin in the measured direction, ie, $|\theta^-\rangle$ At an angle θ From the axis H, which was the original direction, and consists of a superposition of the various spin states in the original (which was) measurement base. x^\pm

Rationale for how to write the situations: Students usually have no problem with the situation. They understand this from the previous line, and from the results of the experiment they received (see the quantitative experiment with the help of the Stern-Gerlach experiment). Given the $|\theta^+\rangle$ To accept, the probability is. Therefore, given, the chance of getting is the complementary probability, i.e., $|x^+\rangle |\theta^+\rangle \cos^2 \frac{\theta}{2} |x^+\rangle |\theta^-\rangle \sin^2 \frac{\theta}{2}$

Hence the coefficient of is (the root of the probability). The minus sign is intended to take care of the orthogonality of the two situations and, that is, to make sure that the overlap between them is zero. It was possible to put the minus even before the coefficient of the cosine, but we chose to mark it this way, because it fits the way we mark the coefficients with polarization of light, and there it is obtained in a really geometric way (see below). $|x^+\rangle \sin \frac{\theta}{2} |\theta^+\rangle |\theta^-\rangle$

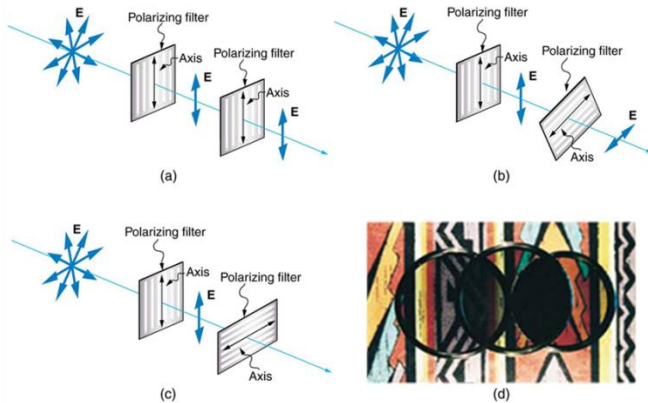
Note that the coefficients in the square express the probability, so the + or - sign in front of them does not affect the probability, which of course is important as stated above, to keep the relative sign between some state members: in one situation it is important that the coefficients' signs are the same and in the other Different. In general, for two situations to be contradictory, the sum of the multiples of their coefficients must be zero.

We can now calculate probabilities for spins in different directions, given a known spin result. We will compare the calculations to the simulator https://phet.colorado.edu/sims/stern-gerlach/stern-gerlach_en.html



Another example: polarization of light

The polarization of a wave is the direction in which the wave oscillates. For example, in a string, whether the wave is standing or moving, the direction of the oscillations, which is perpendicular to the string, is called the polarization direction. Light is an electromagnetic wave, so perpendicular to the direction of light progression is the direction in which the electric field oscillates, and it is defined as the polarization direction (we chose the direction of the electric field and not the direction of the magnetic field. This choice is arbitrary, but is the accepted choice). Assuming that the wave is advancing in the axis direction, then the polarization can be horizontal - in the axis direction x , Or vertical - in the axis direction y



השפעה של שני מקטבים:

(a) האור שעובר את המקטב הראשון מקוטב בכיוון מסוים, ועובר גם את המקטב השני, המונח כך שכיוון הקיטוב שלו מקביל לזה של הראשון.
 (b) כאשר המקטב השני מונח כך שכיוון הקיטוב שלו בזווית ביחס לראשון, רק חלק מהאור עובר, כשהעוצמה נקבעת לפי קוסינוס הזווית בין כיווני הקיטוב.
 (c) כאשר כיווני הקיטוב של שני המקטבים ניצבים זה לזה, אור לא עובר.
 (d) בתמונה זו (credit: P.P. Uroae) מקטב מונח מעל שניים אחרים: כיוון הקיטוב שלו ניצב לכיוון הקיטוב של המקטב הימני (האור החשוך לגמרי) ומקביל לכיוון הקיטוב של המקטב השמאלי (רואים את הרקע דרך שני המקטבים). אמנם העוצמה נחלשת מעט, בגלל בליעה.

The effect of polarization can be seen with the help of a polarizer (polarizer) - A device that allows light to pass only at a certain polarization. We can put the polarizer vertically or horizontally. If we place the polarizer horizontally, then horizontally polarized photons will pass through it, and those that are vertically polarized will not pass through it. Therefore, when two poles are facing each other, all the photons are blocked and light does not pass through the two poles.

We will describe a phenomenon, and then explain it with the help of quantum physics: when two poles are placed opposite each other, the intensity of light varies according to the relative direction between them. When their polarization direction is parallel - the intensity of light passing through them is maximal and when their polarization directions are perpendicular to each other, the intensity of light is zero. Surprisingly, adding a third polarizer, between them at some angle, increases the intensity of the light. This is an intriguing and bizarre experiment.



some angle, increases the intensity of the light. This is an intriguing and bizarre experiment.

The polarization phenomenon is quantum. That is, horizontal polarization and vertical polarization are contradictory situations. Polarization at any angle (relative to the horizontal axis) is a superposition of horizontal and vertical polarization. There are basically two states here; and every other state is a geometric superposition of these two states. We will write down the expressions for the situations and then explain: x, y

$$|\theta\rangle = \cos \theta |x\rangle + \sin \theta |y\rangle$$

$$|\theta^\perp\rangle = -\sin \theta |x\rangle + \cos \theta |y\rangle$$

Explanation: The position at an axis angle is actually a geometric superposition of sine and cosine (like any normal decomposition of a vector). The contradictory state, perpendicular to it, is in fact a state in the addition of, and remember: $\theta + 90^\circ$

$$\cos(\alpha + 90^\circ) = -\sin \alpha \quad \sin(\alpha + 90^\circ) = \cos \alpha$$

If there is any vector at an angle from the positive direction of an axis, then its components will be: $A \cos \theta$ and $A \sin \theta$

$$A_x = A \cdot \cos \theta \quad A_y = A \cdot \sin \theta$$

If we take a vector perpendicular to it (of the same size), i.e. at an angle, then its components will be: $\theta + 90^\circ$

$$A_x^\perp = A \cdot \cos(\theta + 90^\circ) = -A \cdot \sin \theta \quad A_y^\perp = A \cdot \sin(\theta + 90^\circ) = A \cdot \cos \theta$$

From these expressions derives directly the representations to the states we have written to the polarization states. Calculating the overlap will show that these are conflicting situations (i.e., the overlap is equal to zero). $\langle \theta^\perp | \theta \rangle$

It should be noted that the expressions we wrote to describe the polarization are reminiscent of those we wrote in the spin, but in the polarization the angle appears (as we explained in a simple geometric way), and in the spin - half the angle. It has to do with the question of them Contradictory states, whether states with a vertical angle of 90 degrees (polarization) or with the opposite direction, at an angle of 180 degrees from each other (spin).

The following pages offer a quantitative experiment for the examination of polarization. The probability is measured by the intensity of the light. The light intensity is actually proportional to the amount of photons. This amount, stems directly from their probability of crossing the polarizer. The experiment can be performed in several ways, and students should be asked to predict the results before the experiment.

Experimental set-up: three polarizers in a row. The first can be fixed, the second can be rotated in any direction we want (around an axis), and can be removed, the third can be rotated (around the z axis). Light is projected through the polarizers, and its intensity is measured using a light sensor (It is also possible on the mobile phone in a suitable application).

step one: Polarization Calibration.

Place the first polarizer in front of the light source.

1. Measure the maximum intensity of light with a single polarizer.
2. Add the third pole (do not add the second, yet) and find the angle at which the third pole is perpendicular to the first.
3. Find the angle at which the third polarizer is parallel to the first.
4. Add the second polarizer and find the angle at which the second polarizer is perpendicular to the first. Note that one can find the angle at which the second polarizer is perpendicular (or parallel) to the first by direction of the third polarizer parallel to the first, and find the maximum (parallel) and minimum (vertical) intensities depending on the angle of the second polarizer.

second level: Finding the relationship between the probability that the photons will pass (i.e., the illumination intensity) and the relative angle between the first and third polarizers (without The second polarizer) (The resulting relationship is) Find the illumination intensity when there are two

polarizers. $I \propto \cos^2 \theta$ Do it from a situation where the polarizers are parallel to each other and until the polarizers are perpendicular to each other. Measure the relative angle in jumps of 10° .

5. Present the results in a graph of light intensity as a function of the relative angle between the polarizers. Look at a 0 degree angle and a 90 degree angle, and try to guess what the function is in the graph. Add a new variable with which you can construct a linear graph to confirm your hypothesis (hint: remember the experiment we did with the simulation).

Third stage: the effect of the middle polarizer.

6. Place the first and third poles in a vertical position. Find out at what angles the middle polarizer needs to be in order for us to get maximum power? Minimal?
7. Make a graph of illuminance depending on the angle of the middle pole (when the first and third are perpendicular). What is dependency? Explain this theoretically as well.

Answer to the last section (by probability analysis only, and without Dirac markings):

From the first polarizer emit only photons with polarization at the angle of the polarizer - it is called angle θ . The probability of these photons passing the second polarizer is $\cos^2 \theta$. The third polarizer is at an angle from the second polarizer, so the probability of the photons passing the second polarizer passing through the third is as well. Thus, the probability of the photons (passing the first - half of the total photons) passing the second and third polarizers is the product of the probabilities multiplied by: $\cos^2 \theta \cos^2 (90^\circ - \theta) = \sin^2 \theta$

$$\cos^2 \theta \cdot \sin^2 \theta = \frac{1}{4} \sin^2 2\theta$$

This means that we get darkness at angles of (cases where the middle polarizer does not actually affect), and maximum intensity at the angle of $0^\circ, 90^\circ, 45^\circ$

In the lab report, or in a class discussion after the experiment, it is advisable to explain the results while noting the measurements, the collapse of the wave function, and the use of Dirac markings.

Polarization uses can be found at the link: <https://stwww1.weizmann.ac.il/lasers/?p=4803>



Raspberry Activity: Calculations of polarization states using Dirac markings.

Development in time

So far we have talked about states that do not change, but in physics there are dynamics of evolution over time. In classical physics the evolution of time is related to Newton's second law, by which one can find the acceleration, and in any case calculate the velocity and position.

How does time evolution affect a state of quantum physics?

We have seen in the past the relationship between energy and frequency:

$$E = hf = \hbar\omega$$

From this it can be concluded that each energy has a frequency (or angular velocity). In fact, every state of energy has a kind of internal clock that "rotates" at an angular velocity. Suppose that at the time the system was in a state of energy. A clock is attached to this mode at the appropriate frequency. We will mark the situation after a time as follows: $E_i \omega_i = \frac{E_i}{\hbar} t = 0 |\psi_i\rangle E_i t$

$$|\psi_i(t)\rangle = |\psi_i\rangle \cdot \{\omega_i t\}$$

Note that this is not an acceptable notation, but a form of abstraction of the accepted notation and calculation, which include the use of complex numbers.

We will meet the meaning of the accompanying clock when we want to calculate probabilities of situations. When you square the expression, you reach a state of multiplication between two clocks. The product will be defined as follows $\langle \psi | \varphi \rangle$ ¹² Derived from the mathematics of complex numbers:

$$\{\theta_1\} \cdot \{\theta_2\} = \cos(\theta_1 - \theta_2)$$

Remember that, so there is no difference between $\cos(\theta_1 - \theta_2) = \cos(\theta_2 - \theta_1)$ $\{\theta_1\} \cdot \{\theta_2\} \{\theta_1\} \cdot \{\theta_2\}$

What does a state that is a superposition of different energy states look like? Suppose situations with energies. Status: $|1\rangle, |2\rangle, |3\rangle, \dots E_1, E_2, \dots$

$$|\psi\rangle = A_1|1\rangle + A_2|2\rangle + \dots$$

Will develop in time as follows:

$$|\psi_t\rangle = A_1|1\rangle \cdot \{\omega_1 t\} + A_2|2\rangle \cdot \{\omega_2 t\} + \dots$$

In order to understand the meaning of this, we would like to look at a particle that is initially in a certain state, and examine its probabilities of being in one state or another after a while. Let us first look at a state of definite energy (as opposed to a state that is a superposition of states of different energies). What is the probability that after a certain time the particle will be in its initial state? $E = \hbar\omega$

$$\langle \psi | \psi_t \rangle = \langle \psi | \psi \rangle \cdot \{\omega t\} = 1 \cdot \{\omega t\}$$

The probability will be:

$$\langle \psi | \psi_t \rangle^2 = \{\omega t\} \cdot \{\omega t\} = \cos(\omega t - \omega t) = \cos 0 = 1$$

We see that in a state of definite energy, the probability does not change, and the state remains stable without change.

Let us now look at a state that is a superposition of different energies.

$$|\psi\rangle = A_1|1\rangle + A_2|2\rangle + A_3|3\rangle$$

In this case, as we have seen before, the probability of being in a situation is. What will happen to the probability after a while? For this purpose we must calculate the overlap between the state that develops in time and one of the states that make it up, for example. $|i\rangle A_i^2 |2\rangle$

$$|\psi_t\rangle = A_1|1\rangle \cdot \{\omega_1 t\} + A_2|2\rangle \cdot \{\omega_2 t\} + A_3|3\rangle \cdot \{\omega_3 t\}$$

The overlap is:

$$\begin{aligned} \langle 2 | \psi_t \rangle &= \langle 2 | \cdot [A_1|1\rangle \cdot \{\omega_1 t\} + A_2|2\rangle \cdot \{\omega_2 t\} + A_3|3\rangle \cdot \{\omega_3 t\}] = \\ &= A_1 \langle 2 | 1 \rangle \cdot \{\omega_1 t\} + A_2 \langle 2 | 2 \rangle \cdot \{\omega_2 t\} + A_3 \langle 2 | 3 \rangle \cdot \{\omega_3 t\} = \\ &= A_1 \cdot 0 \cdot \{\omega_1 t\} + A_2 \cdot 1 \cdot \{\omega_2 t\} + A_3 \cdot 0 \cdot \{\omega_3 t\} = A_2 \cdot \{\omega_2 t\} \end{aligned}$$

¹² This manner is due to the mathematics of complex numbers, and goes beyond the scope of the discussion in this essay

The probability will be:

$$P = \langle 2|\psi_t \rangle^2 = (A_2 \cdot \{\omega_2 t\})^2 = A_2^2 \cdot \{\omega_2 t\} \cdot \{\omega_2 t\} = A_2^2 \cdot \cos(\omega_2 t - \omega_2 t) = A_2^2$$

The probability we got is the same as it was in the initial state. You can describe what we got as a quantum version of energy conversion. The energy of a particle does not change. Not only does the energy not change, but the probability of being in a certain energy does not change over time.

What, then, does timely evolution mean? For this purpose, we will look at the following example: data two states with defined and different energies, and in two superposition states: $|1\rangle, |2\rangle$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|2\rangle$$

$$|\varphi\rangle = \frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}|2\rangle$$

It is easy to see that the states are contradictory, i.e. given that the particle is in the state, the probability of being in the state is zero. What will happen after a while? Suppose the particle is in a state, what is the probability after a time that the particle is in a state? $\langle\psi|\varphi\rangle\langle\psi|t|\varphi\rangle$

$$|\psi_t\rangle = \frac{1}{\sqrt{2}}|1\rangle \cdot \{\omega_1 t\} + \frac{1}{\sqrt{2}}|2\rangle \cdot \{\omega_2 t\}$$

$$\begin{aligned} \langle\varphi|\psi_t\rangle &= \frac{1}{\sqrt{2}}(\langle 1| - \langle 2|) \cdot \frac{1}{\sqrt{2}}(|1\rangle \cdot \{\omega_1 t\} + |2\rangle \cdot \{\omega_2 t\}) = \\ &= \frac{1}{2}(\langle 1|1\rangle \cdot \{\omega_1 t\} + \langle 1|2\rangle \cdot \{\omega_2 t\} - \langle 2|1\rangle \cdot \{\omega_1 t\} - \langle 2|2\rangle \cdot \{\omega_2 t\}) = \\ &= \frac{1}{2} \cdot [\{\omega_1 t\} - \{\omega_2 t\}] \end{aligned}$$

And the probability will be:

$$\begin{aligned} \langle\varphi|\psi_t\rangle^2 &= \left(\frac{1}{2} \cdot [\{\omega_1 t\} - \{\omega_2 t\}]\right)^2 \\ &= \frac{1}{4} \cdot [\{\omega_1 t\} \cdot \{\omega_1 t\} - \{\omega_1 t\} \cdot \{\omega_2 t\} - \{\omega_2 t\} \cdot \{\omega_1 t\} + \{\omega_2 t\} \cdot \{\omega_2 t\}] = \\ &= \frac{1}{4} \cdot [1 - \cos(\omega_1 t - \omega_2 t) - \cos(\omega_2 t - \omega_1 t) + 1] = \\ &= \frac{1}{4} \cdot [2 - 2\cos(\omega_1 t - \omega_2 t)] = \frac{1}{2} \cdot [1 - \cos(\omega_1 t - \omega_2 t)] \end{aligned}$$

The result is the magnitude of the variable in time, when its variable range is between 0 and 1, i.e. any probability is possible, and is reached at a certain time. That is, at every moment (except for specific time points) there is a different probability from zero of being in a state that contradicts the initial state.

As an interim summary of the issue of evolution in time, we will now look at the structure of periphery-body-nucleus.

Periphery: Dynamics and the evolution of a time state in classical physics derives from Newton's second law (from which we can find the acceleration and in any case know how the state changes in time). In addition, there is of course an energy conservation law.

Nuclear: In quantum physics each state is accompanied by an internal clock, the frequency of its rotation is affected by the energy of the state. The quantum formulation of the law of conservation of energy is that the self-state of energy does not change, moreover, even in superposition states the probabilities of being in one energy or another do not change (and thus there is actually no change in energy). What does change? Probabilities for other physical quantities or probabilities of being in one state or another (provided they are not self-states of energy).

Body: All the examples we solved and solved.

Exercise - timely development in a system with 3 modes

Note: Solving this exercise requires a number of mathematical calculation steps.

Given a system with three modes. The marking of any condition is $|n\rangle$, When, and the energy of such a state $h\nu_n = 1, 2, 3$ $h\nu_n = n \cdot \epsilon$ (The marking is a marking only for any size having units of energy). ϵ

Data two modes:

$$|\psi_1\rangle = \frac{1}{\sqrt{2}} |1\rangle + \frac{1}{\sqrt{3}} |2\rangle + \frac{1}{\sqrt{6}} |3\rangle$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}} |1\rangle - \frac{1}{\sqrt{3}} |2\rangle - \frac{1}{\sqrt{6}} |3\rangle$$

- α. Show that the sum of the coefficients of the coefficients in each of the states is 1 (i.e., it is a physical state that is written correctly).
- β. Show that the two situations are contradictory, that is, given that the situation is, there is a zero probability of getting a measurement that will say that the situation is. $|\psi_1\rangle|\psi_2\rangle$
- γ. Given that the system has started in mode. When will the system be in place? In other words, when is the chance of being in a situation 100%? If it is not possible to be in a situation, what is the highest probability for this situation to be reached? when will it happen? $|\psi_1\rangle|\psi_2\rangle|\psi_2\rangle|\psi_2\rangle$

Guidance for section C

First, think about. Remember that, you can mark: $|\psi_{1t}\rangle \omega = \frac{E}{h} = n \cdot \frac{\epsilon}{h}$

$$\omega_1 = \frac{\epsilon}{h} \quad \omega_2 = 2\omega_1 \quad \omega_3 = 3\omega_1.$$

Next think about. $\langle\psi_2|\psi_{1t}\rangle$

Next we calculated the, as we defined the product $\langle\psi_2|\psi_{1t}\rangle^2 \{\theta_1\} \cdot \{\theta_2\} = \cos(\theta_1 - \theta_2)$.

Enter similar organs (do not forget that). $\cos(\theta_1 - \theta_2) = \cos(\theta_2 - \theta_1)$

We want it to take place. $\langle\psi_2|\psi_{1t}\rangle^2 = 1$

You got an equation (trigonometric ...) - solve it!

Hints for solving the equation:

Help with identity. $\cos 2\alpha = 2 \cos^2 \alpha - 1$

Define. $x = \cos \omega_1 t$

You got a quadratic equation - solve it !!

If you solved, find the (do not forget to work in radians !!!!). t

If there is no solution to the equation, that means there can be no situation in it. We will try to see what the probability is $\langle \psi_2 | \psi_{1_t} \rangle^2 = 1$ The maximum P for which we can get a solution to the equation. $\langle \psi_2 | \psi_{1_t} \rangle^2 = P$

Reminder: The number of solutions to a quadratic equation of shape depends on the size of the discriminant, i.e. the expression. If it is positive there are two solutions, if it is zero there is a single solution, and if it is negative there is no solution. If we write for our equation the, it is easy to see that as we decrease the probability $ax^2 + bx + c = 0$ $\Delta = b^2 - 4ac$ ΔP we ask, so it will grow. Thus, the highest probability you will give a solution will be for $\Delta = 0$

Now find the maximum probability to get the measurement in, and the time that will elapse until we reach that probability. $|\psi_2\rangle$

Mathematical Introduction to the Schrödinger Equation

Operators and self-modes

When a state is described by some function, we want to know the value (or possible values) of one physical quantity or another. In quantum physics, physical quantities are represented by operators. What is an operator? An operator is a mathematical creature that performs some action on a function. This can be a number multiplication, a function multiplication or anything else. We will also meet shear operators, i.e. operators whose operation on the function is a derivative operation. The operator is usually marked with a hat. Examples of operators:

$$\hat{A}f = 2f \quad , \quad \hat{A}f = x \cdot f \quad , \quad \hat{A}f = f' \quad , \quad \hat{A}f = f''$$

Sometimes the function expresses a self-state of a certain physical quantity, i.e. a state in which that physical quantity has a single value and not a superposition. In other cases the function will express superposition modes. In the case of a self-state then operating the operator on the function will not change the function but will multiply it by the value given to the relevant physical size. This value is called the operator's self-value, and the function is called its self-function.

For example (so far only mathematically, without the physical context):

We will take the shear operator. It is easy to see that the function is a self-function of the operator, since: $\hat{A}f = f'f(x) = e^{\alpha x}$

$$\hat{A}f = f' = \alpha \cdot e^{\alpha x} = \alpha \cdot f$$

Running the operator on this specific function gave a doubling of the function in a fixed number. Therefore this function is a self-function of the operator. If this operator expresses a physical quantity we can say that the function expresses a self-state of the same physical quantity (i.e. a state with a single value and not a superposition), and the value of that physical quantity is of course α .

Note: This was a mathematical example only. In fact, the shear operator by itself does not represent a physical quantity.

Additional note: We have not shown, nor will we see below, how self-function can be found in general. We will always be content with presenting the self-function, and convincing that it is indeed so.

Differential Equations

The characteristic of most of the equations we know is that solving the equation is a particular value of the invisible. For example, the solution of the equation, is the value. A differential equation is a different type of equation. In this equation the function and its derivatives appear, with such and such mathematical operations, and the solution of the equation is not a particular value, but the function itself. $2x + 3 = x + 1x = -2$

For example, let's take the equation:.. The solution to the equation is of course the family of functions: $f' = x + 2$

$$f = \frac{x^2}{2} + 2x + c$$

We will not learn here how to solve any differential equation, but rather give a limited number of simple examples that are easy to be convinced of their correctness.

Let us look at the following differential equation:

$$f' = af$$

We have seen earlier this example in the context of operator and self-function.

The solution to the equation is as follows: $f = B \cdot e^{ax}$

When representing some fixed quantity [it should be emphasized to the students the difference between the fixed number, which has a specific value in each equation, and the fixed number, which can be any number, i.e. there are infinite solutions here]. For if we cut we will get: BaB

$$f' = Bae^{ax} = af$$

Let us now look at the following equation:

$$f'' = af$$

As we will see immediately, in this equation there is a difference between the case where it is positive and the case where it is negative. To emphasize this, we will split it into two cases with the following notation: a

$$f'' = b^2 f \quad , \quad f'' = -b^2 f$$

In the first case, the function satisfies the equation (check it out!), As well as the function satisfies the equation. Thus, the general solution is any superposition of the two solutions: $f'' = b^2 f$ $f = Ae^{bx} + Be^{-bx}$

$$f = Ae^{bx} + Be^{-bx}$$

After all:

$$f' = bAe^{bx} + (-b)Be^{-bx}$$

$$f'' = b^2Ae^{bx} + (-b)^2Be^{-bx} = b^2(Ae^{bx} + Be^{-bx}) = b^2f$$

Note: A distinction must be made between the constant that is given in the equation, and the constants that are free to be any number (similar to the set in the integral). bA, Bc

In the second case $f'' = -b^2f$ The solution is different. Do we know a function whose second derivative is the function itself with a negative coefficient? There are two functions that do this: $\sin bx$, $\cos bx$. The general solution is all their superposition: $\sin bx$, $\cos bx$

$$f = A \cdot \cos(bx) + B \cdot \sin(bx)$$

After all:

$$f' = -A \cdot b \cdot \sin(bx) + B \cdot b \cdot \cos(bx)$$

$$f'' = -A \cdot b^2 \cdot \cos(bx) - B \cdot b^2 \cdot \sin(bx) = -b^2f$$

Schrödinger equation

In classical physics, the motion of a particle is described using an equation of motion, which usually derives from Newton's second law. In quantum physics we attribute to a particle the properties of a wave, so instead of an equation of motion we need a wave equation. A wave equation is an equation whose solution will give us the wave function that describes the state of the particle (the different probabilities of being in different places or in different situations). This equation in quantum physics is called the Schrödinger equation. We would like to find her.



In the waves of de Broglie we saw that there is a relationship between the momentum of a particle and its wavelength.

That is, there is a relationship between the particle property (momentum) and the wavy property (wavelength). The connection is. This means that when the wave function has a defined wavelength (sine or cosine function) there is a defined momentum $p = \frac{h}{\lambda}$. If the wave function is a different function, then in mathematical ways [advanced, far beyond our ability at present] it can be described as a sum of sines and cosines, i.e. it is a superposition of different wavelengths, and in any case as a superposition of different motions. Let us take a state with a definite momentum, i.e. a cosine function (in the same way a sine function can be taken).

$$\Psi = A \cdot \cos(kx)$$

¹³In fact, as has already been emphasized, a wave of a sine or a cosine does not have a single momentum but a superposition of two motions of the same size and opposite in their direction. The exact presentation of things is the composite presentation. We avoid this in order to make it accessible to students. It is worth emphasizing to the students that there are actually two motives here, but they are of the same size.

As some numerical coefficient. We do not currently discuss its meaning.

what is k ? Is called the spatial frequency of the wave, and it expresses how dense the oscillations of the wave are¹⁴. The larger it is, the denser the wave oscillations will be. We will try to understand its meaning with the help of placement $x = \lambda$, This is where the wave ended a single wavelength, so the expression inside the cosine is equal to. That is: 2π

$$k\lambda = 2\pi$$

We will present the De Brulee formula here and we will get: $\lambda = \frac{h}{p}$

$$k \cdot \frac{h}{p} = 2\pi$$

That is: $k = \frac{p}{h} \cdot 2\pi = \frac{p}{\hbar}$.

We used the marking here, which symbolizes the plank constant divided by, which is a more useful size than the regular plank constant. $\hbar = \frac{h}{2\pi}$

Anyway, according to the context, we can now write the wave function in the form: $k = \frac{p}{\hbar}$

$$\Psi(x) = A \cdot \cos\left(\frac{p}{\hbar} \cdot x\right)$$

We will derive the wave function.

$$\Psi'(x) = -\frac{p}{\hbar} \cdot A \cdot \sin\left(\frac{p}{\hbar} \cdot x\right)$$

We will cut a second time.

$$\Psi''(x) = -\frac{p^2}{\hbar^2} \cdot A \cdot \cos\left(\frac{p}{\hbar} \cdot x\right) = -\frac{p^2}{\hbar^2} \cdot \Psi(x)$$

That is:

$$\Psi'' = -\frac{p^2}{\hbar^2} \cdot \Psi$$

Or otherwise:

$$\widehat{p^2}\Psi = -\hbar^2\Psi''$$

The momentum squared can be treated as an operator cutting¹⁵.

The energy is of course a sum of the kinetic and potential energy:

¹⁴ k is used in a role similar to, where the function is of location and not of time.

¹⁵ We are talking about a squared start-up operator, not a start-up operator, in order to avoid the need for complex numbers.

$$E_k + U = E$$

$$\frac{1}{2}mv^2 + U = E$$

$$\frac{1}{2}m\left(\frac{p}{m}\right)^2 + U = E$$

$$\frac{p^2}{2m} + U = E$$

If we refer to momentum squared as the shear operator we saw, and "activate" the two sides of the equation on the wave function we get the equation:

$$-\frac{\hbar^2}{2m}\Psi'' + U \cdot \Psi = E \cdot \Psi$$

This equation is called "Schrödinger equation", And is as stated the central equation in quantum physics. This equation is a wave equation, and is Differential equation. A differential equation is an equation in which a function and its derivatives appear, where the solution of the equation is not to find one value or another of the variable, but the function itself. As stated, if we assume that the wave equation is a sine or cosine, we get that it has a defined wavelength, i.e. a defined momentum. Otherwise, the function will describe a state of superposition of a number of motives.

A distinction must be made between the wave equation - the Schrödinger equation, and the wave function, which is in fact the solution of the Schrödinger equation.

More examples:

- Particle in a box (sinuses and quantization)
- A free particle and a particle in a forbidden place - a tunnel

Particle in a box

A particle with mass can move in one dimension in the region $0 < x < L$. Is actually the length of the (one-dimensional) box in which the particle is trapped. No additional forces are acting so there is no potential energy here. The Schrödinger equation takes shape:

$$-\frac{\hbar^2}{2m}\Psi'' = E\Psi$$

That is:

$$\Psi'' = -\frac{2mE}{\hbar^2}\Psi$$

The energy is positive (since there is only kinetic energy). We have already seen such an equation, and the solution to the equation is as follows:

$$\Psi = A \cos\left(\frac{\sqrt{2mE}}{\hbar} \cdot x\right) + B \sin\left(\frac{\sqrt{2mE}}{\hbar} \cdot x\right)$$

This is a case where the particle cannot come out of the box, and therefore cannot pass through the edges. The wave function should be continuous and therefore should exist:

$$\Psi(x = 0) = \Psi(x = L) = 0$$

The condition requires that the function be composed of sines (resets to zero) and not cosines. That is, therefore: $\Psi(x = 0) = 0$ and $\Psi(x = L) = 0$

$$\Psi = B \sin\left(\frac{\sqrt{2mE}}{\hbar} \cdot x\right)$$

Let us now consider the condition. In order for the sine function to be reset, the expression within the sine (for the value) must be an integer double of: That is: $\Psi(x = L) = 0$ $x = L$

$$\frac{\sqrt{2mE}}{\hbar} \cdot L = n\pi \quad n = 1, 2, 3 \dots$$

We have ignored negative seas here, since they only add a minus to the wave function, which is meaningless. We ignored the case of $n = 0$ Since it completely resets the function of the wave, that is, there is no place where the particle can be - there is no particle.

In any case, it is obtained:

$$\frac{\sqrt{2mE}}{\hbar} = \frac{n\pi}{L}$$

That is:

$$\Psi_n = B \sin\left(\frac{n\pi}{L} \cdot x\right)$$

It is also accepted that only certain energy levels are allowed:

$$E_n = \frac{\hbar^2 \pi^2}{2mL^2} \cdot n^2$$

when are $n = 1, 2, 3, \dots$

We got quantization in energy levels. Some lone energy levels are allowed, and others are not allowed. In fact, in any physical state where the particle is limited (at least classically) to be in a particular region (such a state is called a related state) we will get quantization in energy levels. So in this case, so in the quantum harmonic motion, and so in the hydrogen atom, as we saw in the Bohr model.

In this context the Bohr model (which is part of the so-called "old quantum theory") can be seen as a bridge between classical physics and quantum physics. This model comes with a deterministic approach like classical physics. In other words, according to this model there is a defined and unique radius for each energy level, there is no uncertainty, and the electron has no wave function. On the other hand, this model has quantum properties, chief among them quantization¹⁶. In this respect a Bohr model is found in the periphery of classical physics, but also in the periphery of quantum physics, as befits a theory whose main importance is historical in the transition to a different understanding of nature.

¹⁶Interesting is the fact that the phenomenon of quantization - the appearance of certain sizes in discrete portions, actually gave quantum physics its name. We emphasize that this phenomenon also exists in classical waves, such as in the string of a guitar. In each string there are only certain possible frequencies.

Particle in a free place and particle in a forbidden place

If we take a free particle - no forces and no potential energy, the Schrödinger equation takes the simple form:

$$-\frac{\hbar^2}{2m}\Psi'' = E \cdot \Psi$$

That is:

$$\Psi'' = -\frac{2mE}{\hbar^2} \cdot \Psi$$

We have already seen this equation, and its solution is:

$$\Psi = A \cos\left(\frac{\sqrt{2mE}}{\hbar} \cdot x\right) + B \sin\left(\frac{\sqrt{2mE}}{\hbar} \cdot x\right)$$

Since there is only a sine and a cosine here, with the same wavelength, we can conclude from this that we have a single wavelength, i.e. one definite momentum (in fact there are two motions here identical in size and opposite in direction, but we will ignore this for now). We have already seen that in the case of a momentum defined the function of the wave is in the form of, so it can be concluded that in our case. This result makes sense, since there is no potential energy and therefore: $\cos\left(\frac{p}{\hbar} \cdot x\right) p = \sqrt{2mE}$

$$\sqrt{2mE} = \sqrt{2mE_k} = p$$

Here we relied of course on the (classic) identity $E_k = \frac{p^2}{2m}$

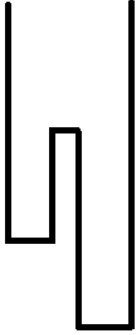
We would now like to look at an energy particle that is in an area where there is potential (constant) energy that has grown so that it exists. We would like to understand the meaning of this first. This is similar to an energy body, which is located in an area where there is a wall that is high, that is, the potential energy is, but the energy of the body is smaller than this size! It is clear and simple that the body cannot be there. But not everything that is simple and clear to us does exist in quantum physics. $E < U$

- Show that in this case (potential energy is constant, and exists), the wave function satisfies the Schrödinger equation. $E - U < 0 \Psi(x) = e^{-bx}$
- Express yourself with the help of problem data. Note that the value of increases as the gap between energy and potential energy increases. b, \hbar, m, E, U

Note: The wave function is not a sine or a cosine, that is, the wave function does not have a single wavelength, meaning it is a case of superposition of motives.

The result we obtained is that the particle has a non-zero wave function in the "forbidden" region. Note that this is a descending exponential function, meaning that there is a decay of the wave function here. A particle that comes from the permissible region - it has a defined wavelength, ie a defined momentum, but in fact it has two motions that are the same size and opposite in their direction. Thus, when it hits a "wall" it will most likely return (in a sort of elastic collision) in the other direction. However, there is a different probability from zero that he will be able to cross the

wall. The thicker the wall, the more the wave function fades and the particle will have difficulty moving to the other side of the wall. However, if the wall is not too thick, the particle can pass to the other side, which means that there is some probability that it will be in the forbidden area ("the wall") and through it will pass to the other side. This phenomenon is called a tunnel - supposedly the electron dug a tunnel and passed through it to the other side.



There are many applications to the tunnel phenomenon. One of them is related to radioactive decay. A radioactive state is an unstable state. In fact, it is partially stable - a meta-stable state. This means that in terms of energy it is in a kind of pit that cannot come out of it, and in this respect it is a stable state. However, there is a relatively small barrier here, which has a quantum probability to cross it. If it passes it, the radioactive decay will take place. It is the quantum probability that makes this process statistical.

We have seen that the starting momentum operator is: $\widehat{p^2}\Psi = -\hbar^2\Psi''$

We have seen that the self-state of a particular operator is a state described by a function, which when the operator operates on it will actually double the function to a fixed size, which is the value of the physical size. We will now look at two modes (described not by Dirac markings but by a wave function), and asked whether they are single-square momentum modes. Even if we get a positive answer, we will not be able to know in this context whether there is a single momentum or a superposition of two motives of the same size and in opposite directions. If we get a negative answer, surely there is a superposition of motives here.

Let's look at the case. The first derivative will be: $\psi(x) = e^{-\alpha x^2}$

$$\psi'(x) = -2\alpha x e^{-\alpha x^2}$$

The second derivative will be:

$$\psi''(x) = -2\alpha e^{-\alpha x^2} + 2\alpha x \cdot 2\alpha x e^{-\alpha x^2} = (4\alpha^2 x^2 - 2\alpha) \cdot e^{-\alpha x^2}$$

And therefore:

$$\widehat{p^2}\psi(x) = -\hbar^2\psi''(x) = -\hbar^2(4\alpha^2 x^2 - 2\alpha) \cdot e^{-\alpha x^2} = -\hbar^2(4\alpha^2 x^2 - 2\alpha) \cdot \psi(x)$$

It's easy to see that turn on A. The operator on the wave function gave the wave function an indefinite coefficient, i.e. it is a case in which there is no single momentum but superposition. $-\hbar^2(4\alpha^2 x^2 - 2\alpha)$

Let us now look at the case. The first derivative will be: $\psi(x) = e^{\beta x}$

$$\psi'(x) = \beta \cdot e^{\beta x}$$

The second derivative will be:

$$\psi''(x) = \beta^2 \cdot e^{\beta x}$$

And therefore:

$$\widehat{p^2}\psi(x) = -\hbar^2\psi''(x) = -\hbar^2\beta^2 \cdot e^{\beta x} = -\hbar^2\beta^2 \cdot \psi(x)$$

It is easy to see that operating the operator on the wave function has given the wave function a constant coefficient, i.e. ostensibly this is a case where there is a single momentum. However, we note that the coefficient is negative, meaning it cannot constitute the value of the momentum squared. Therefore, this case is also a case of superposition. In fact, the only case where we can see a single momentum (square) is a sine or cosine function, or an exponential function with a simulated estimator (composite numbers), which is actually equivalent to sine and cosine. $-\hbar^2 \beta^2$

Worksheet

Answer the following questions (At the end of the worksheet you will find a "formulas page"). If the answer is clear to you without calculation (or with the help of a very, very immediate calculation), you can answer it without calculation. It is always possible (and recommended!) To back yourself up As In calculating whether this can be done.

1. Given a system with the (contradictory) situations $|0\rangle, |1\rangle, |2\rangle$.

The energies of the states are: $E_0 = 0$, $E_1 = E$, $E_2 = 2E$

Data for the following situations:

$$|a\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|1\rangle + \frac{1}{2}|2\rangle$$

$$|b\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{2}|1\rangle - \frac{1}{2}|2\rangle$$

$$|c\rangle = \frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}|2\rangle$$

1. Given that the system is in state, what is the probability that the measurement result will say that we: $|a\rangle$
 - In mode $|0\rangle$?
 - In mode $|1\rangle$?
 - In mode $|2\rangle$?
 - In mode $|b\rangle$?
 - In mode $|c\rangle$?
2. Given that the system is in state, what is the probability that the measurement result will say that we: $|a\rangle$
 - In mode $|0\rangle$?
 - In mode $|1\rangle$?
 - In mode $|2\rangle$?
 - In mode $|a\rangle$?
 - In mode $|c\rangle$?
3. Are the situations contradictory? $|a\rangle, |b\rangle, |c\rangle$
4. Given that the system is in state in time. Wrote $t = 0 |a\rangle$ and An expression (as simplistic as possible!) Of the probability of being in a state as a function of time $|1\rangle t$.
5. Given that the system is in state in time. Wrote $t = 0 |a\rangle$ and An expression (as simplistic as possible!) Of the probability of being in a state as a function of time $|b\rangle t$.

6. Given that the system is in state in time. Wrote $t = 0|b\rangle$ and An expression (as simplistic as possible!) Of the probability of being in a state as a function of time. Is there a time when the probability is 100%?
2. We will now deal with spins (of electrons).
- A figure in the measurement found that the spin in the axis direction is positive. What are the chances of finding a positive spin in the following axes:
 - axis z
 - Axis at an angle of axis 15° .
 - Axis at an angle of axis 45° .
 - Axis at an angle of axis 150° .
 - Given that we are in a situation. What are the chances of finding a negative spin in the following axes: $|\psi\rangle = \frac{1}{\sqrt{3}}|x^+\rangle + \sqrt{\frac{2}{3}}|x^-\rangle$
 - axis z
 - Axis at an angle of axis 15° .
 - Axis at an angle of axis 45° .
 - Axis at an angle of axis 150° .
 - Given that we are in a situation where the probability of getting a positive spin in the axis direction is 25%, and in the wave function there is a minus on the limb of the negative spin. Write down the wave function and find the axis for which there is a 100% probability of finding a positive spin.
 - As is well known, the potential energy of a mass attached to a spring is when the spring is constant. Given a quantum particle with a mass attached to a fixed spring. Given that the wave function of the particle is. (Note the minus). $U = \frac{1}{2}kx^2$ $\Psi(x) = e^{-\frac{\sqrt{km}}{2\hbar}x^2}$
Find the energy of the particle using the Schrödinger equation.
Hint: Work with derivatives

Formulas

Two-mode wave function $|\psi\rangle = A|a\rangle + B|b\rangle$

probability $P = \langle\varphi|\psi\rangle^2$

Probability of one of the superposition modes $P_a = A^2$

Development in time $|\psi_t\rangle = |\psi\rangle \cdot \{\omega t\}$

Angular velocity of position $\omega = \frac{E}{\hbar}$

Multiplication between clocks $\{\phi_1\} \cdot \{\phi_2\} = \cos(\phi_1 - \phi_2)$

Positive spin mode on a pivot angle θ $| \theta^+ \rangle = \cos\frac{\theta}{2}|x^+\rangle + \sin\frac{\theta}{2}|x^-\rangle$

Negative spin spin mode at an angled angle θ $| \theta^- \rangle = \sin\frac{\theta}{2}|x^+\rangle - \cos\frac{\theta}{2}|x^-\rangle$

Schrödinger equation $-\frac{\hbar^2}{2m}\Psi'' + U\Psi = E\Psi$

Chapter 4 - Quantum Communication and Encryption

In May 2018, the Wolf Prize for 2018 was awarded at the Knesset. The Wolf Prize, awarded in Israel, is considered the second most important prize in the world, after the Nobel Prize. About a third of the Wolf Prize winners (in areas where there is a Nobel Prize) later also won the Nobel Prize. In 2018 he won the Wolf Music Award for legendary searcher Paul McCartney. Prof. Wolf Prize in Physics Charles Bennett and Prof. Jill Bersard. They received the award for their contribution to quantum encryption, in particular for the protocol named after them BB84 (since 1984).

We will describe here the process - quantum key transfer.

We will first get to know the characters: in matters of encryption we have two people who want to convey messages to each other, the first is called Alice and the other Bob (although there were those who suggested in Hebrew Aya and Boaz or Arik and Benz). The problem is that Eve wants to eavesdrop on them (farm in English = Eve. The verb "eavesdrop" is Eavesdropping, and the beginning sounds the same).

One of the most powerful encryption ways to send a message between two people is a "one-time notebook" cipher. According to this method, a random cipher is compiled that is at least as long as the message, with the help of which the visible message is converted into an encrypted message. Using the same random cipher, you can also decrypt the message on the other side, and turn the encrypted message into a visible message. That is why this code is also called a "key". The big problem: how to move the key so that it is both in the possession of the sender of the message and in the possession of the recipient - without anyone intercepting it on the way and being able to eavesdrop. Suppose Alice attached some random cipher, and sent it on a note written in red ink on pink paper falling into the water (like the cipher books of the "Enigma") via a Lvov messenger - Eve can catch the messenger, steal his note (before wetting it), Copy the cipher (and keep passing the note to Bob, without Alice and Bob knowing!), Listen to the encrypted message, and translate it with the key to a visible message.

Another problem: the power of the method is only in one-time use of the random cipher. Every time you want to send a message - you have to send a new messenger with a new key, and this is

a very serious weakness (which has dropped a lot of secret messages throughout history). If so, changing keys is a challenge.

Alice wants to safely move Lviv to a key that will serve both of them. It's some password. The password does not matter, it can be random. What matters is that they both knew what the password was and on the other hand, Eve would not know. If the key falls into the hands of Eve, Alice and Bob will want to know about it, and find another password.

The method goes like this:

Alice gives Bob a stream of photons. For each photon it measures the polarization at one of the bases:

1. Horizontal / vertical base, ie checking whether the polarization is in the axis direction x (horizontal -) or y-axis (vertical |). We will mark this test with the + sign. Horizontal result - will be marked 1, and vertical result | Will be marked 0.
2. Diagonal base, i.e. checking whether the polarization is at an angle above the axis 45° x (direction /) or at an angle below the x-axis (direction \). We will mark this test by marking X. Result / will be marked 1, and result \ will be marked 0.

Alice randomly selects the test base. Bob gets the photon and checks its polarization. Bob also randomly selects the base of the test.

Recall the polarization states. Horizontal polarization is and vertical polarization is, and they are of course conflicting situations. The diagonal polarization states can be written as superpositions of the vertical and horizontal states. We will record this as follows: $|x\rangle|y\rangle$

$$| \nearrow \rangle = \frac{1}{\sqrt{2}} |x\rangle + \frac{1}{\sqrt{2}} |y\rangle$$

$$| \searrow \rangle = \frac{1}{\sqrt{2}} |x\rangle - \frac{1}{\sqrt{2}} |y\rangle$$

From the square of the coefficients it is easy to see that each measurement at the base is different from the current situation (determined by the last measurement) gives a chance Of 50% For each of the results.

We will note here that it is also possible to write the superposition modes with slight modifications, such as sign replacements and the like. Since writing the states is only a mathematical tool on the way to calculating probabilities, there is a certain freedom in defining them (for example, doubling a state in will not change the probability which is the square overlap). In a sense it is reminiscent of potential energy, which is a tool on the way to calculation, and since what is important to us there are the differences, we have freedom in defining a plane of reference.—1

There is an important point here that needs to be emphasized to students. Physics is generally perceived by students as an exact science that deals with truths, and so it is indeed strange that there is more than one truth: a number of potential energies or a number of different wave

functions. However, if one sees physics as a tool for understanding the world or a tool for predicting, one can have the freedom we have in the various presentations.

After launching the photons and measuring them, Alice and Bob compare the bases on which they tested each of the photons. The comparison can be made in any communication channel, and does not need to be encrypted. In comparison, they do not tell each other what the test results were (ie, 0 or 1), but only on what basis each test was performed (ie, vertical / horizontal or diagonal).

For the photons measured by both on the same basis, both obtained the same result with certainty. Because as soon as Alice measured the polarization at a certain base, the wave function of the photon collapsed to have a self-state of that particular measuring base. If Bob measures at the same base, he must get the same result that Alice got.

For photons measured at different bases, no matter what the result Alice got, 1 or 0, Lvov has a chance Of 50% Get 1 And a 50% chance of getting 0.

Now, Alice and Bob can take the measurement results made on the same base (the measurement results made on different bases do not interest them and will not be part of the encryption key, so they can ignore these results (delete them from the list of 1s and 0s they have)). They both have the same sequence of unity and zeros. This sequence can be used as the password agreed upon by both, that is, for the encryption key of the message.

We will demonstrate this using the following table. It describes the bases on which each is measured, and the results of the measurements. The code will be determined by the measurements made on the same basis.

The direction Alice was measuring	X	X	+	X	+	+	+	X	+	X	X	X	+	+	X	+	X	+	+	X
Alice's result	\	/		/	-		-	\	-	\	/	\		-	/		/		-	/
The direction in which Bob measured	+	X	+	+	+	X	X	X	+	+	X	+	+	+	+	+	X	X	+	+
Bob's result	-	/		-	-	/	\	\	-	-	/			-	-		/	\	-	
The character that will enter the code		1	0		0			0	1		1		0	1		0	1		1	

What is the advantage of this method? Using this method you can tell if someone is eavesdropping, thus making sure that no one but Alice and Bob knows what the code is. why? Eve, the eavesdropper, could only measure the polarization each time on a random basis. Alice and Bob did not coordinate the bases in advance, so she was unable to know in advance the bases of each and every measurement. If we take the photons that Alice and Bob measured on the same basis (remember, the rest of the photons are not of interest to us), then in the probability that half of them Eve measured on a different basis than what Alice and Bob measured. The problem is that its measurement affected the situation. If for example Alice measured at base + and got 1, when Eve measures at base + she will also get 1, and when Bob measures at base + he will also get 1. In this case, since Eve measured at the base of the photon's self-state (created after Alice's measurement), then That the measurement only describes the situation to her and does not change it. However, if for example Alice measured at base + and got 1, if Eve will measure at baseX (diagonal) it will get 0 or 1 in equal odds. No matter what Eve gets, in either case 0 or 1, a bob that gets the photon after Eve eavesdropped on it, can get a 0 or 1 at equal odds (remember, Bob measures at Alice's base and not Eve's). In other words, if Eve is eavesdropped on, there is a 50%

chance that Bob will get a different result than Alice. Alice and Bob can compare in a visible channel some of the results (measured on the same basis) for the purpose of checking whether there are eavesdroppers or not. Of course, they will not compare everything, because they may be eavesdropped on for comparison, so they will only compare a particular sample.

We will emphasize that even if Eve then knows what the correct bases were (because Alice and Bob talk about it on an unencrypted channel), the situations have already been changed by her so that Alice and Bob will know of her existence.

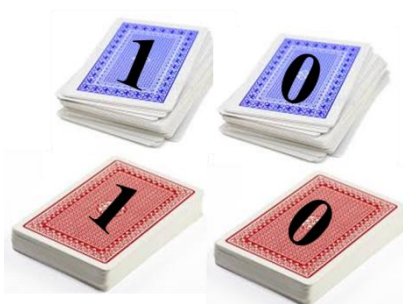
Women note that about half of the photons were measured by Alice and Bob at the same base, and half at different bases. That is, the photons of interest to them are about half of all the photons sent.

Out of those photons, if indeed Eve is eavesdropping, then in half of them she measures on the same base as Alice and Bob, so she does not affect them. Only in the half in which it measures at another base does it cause the wave function to collapse. Of these, in half Bob will get the same result as Alice, and in half he will get a different result than Alice.

In total, in the case of eavesdropping, in a quarter of the relevant photons (one-eighth of all photons sent) Alice and Bob will be able to see traces of Eve. As mentioned, all the odds we have mentioned are probabilistic, and there can be considerable exceptions if it is a small number of photons. However, for large numbers, the odds of counting are very accurate.

For example, if Alice launches 1024 photons, about 512 of them are measured by Alice and Bob at the same base. They can take 256 of them for control and 256 of them as a password. If Eve eavesdrops, then in about a quarter of the 256 control photons, that is, in about 64 photons, Alice and Bob will get different results. Such a result will indicate that there is eavesdropping. Of course, 1024 is a small number of photons, and you can easily send a much larger number.

Suggestion for illustration activity:



Make notes. Each note has two sides - one blue and the other red. On each side of the note is written 0 or 1, so there are four types of notes: two sides 1, both sides 0, blue 0 and red 1, blue 1 and red 0. The number of notes of each type should be the same (more or less). The colors represent the measurement bases, and the numbers the result.

Mix all the cards well. The class is divided into three groups: "Alice", "Bob" and "Eve". Alice's team gets half of the notes, and Eve's team gets the other half. The cards that Eve has are arranged as follows: Half of them are sorted according to the values of the blue side: 0/1, and half of them according to the values of the red side. This creates a situation where Eve has 4 groups of cards, in each group one side is the same (for example: red 0) and the other side consists of two options at random (in this case: blue 0/1). See drawing: On the other side the group does not know what appears.

The group Alice randomly takes a note from the notes she has, chooses one side (color) and writes down the color and number and passes the note to Eve. Eve can choose one of the following two actions, but she must maintain consistency throughout the experiment:

1. Be a loyal courier, and just hand over the notes to Lviv.
2. Eavesdrop on the call. In this way she has to look at only one side of the note (for example, the red side), write down its details (color and number), and pass another note to Lvov instead. The other note that the group Eve passes must be the same as the side she chose to look at, that is, if she looked at the red side and had the number 1 on it, she must pass a note from the pack on the red side has the number 1. She must not look at the other side and / or choose a note According to him.

Bob, who receives the notes from Eve, has to choose a side (i.e., choose a color), and write down the result obtained (color and number). Many notes are repeated like this. It is very important to work neatly.

After Alice finishes all her notes, Alice and Bob compare the colors (the colors only!). All the cases where Bob chose a color that is not the same as Alice's color, set aside. Alice and Bob choose a particular sample of notes for control. The control notes they have to find out if Eve was eavesdropped on, or if Eve was a loyal messenger, and then they have to create (each group individually) the code, and find out that they did indeed receive the same code.

Note: The more notes there are the better we can see the statistical odds. It is likely that even with a relatively small number of notes we will be able to identify if Eve is eavesdropping, but see chances according to the probabilities we said - for this you need a relatively large number of notes, which may take a long time for the experiment process.

Another version: In order to prevent the inconsistency of the Eve group from destroying the results, it is possible that the teacher will serve as the Eve group. In this way the game is conducted as a competition between the students and the teacher, the task of the students: to identify eavesdroppers or alternatively to create a uniform code. The advantage of the version is that Eve will be consistent. Its disadvantage is that the students do not experience Eve's action (which is significant for understanding).

Exercises

1. According to what we have read, the polarization of photons is used to perform the process. Why use photon polarization rather than electron spin (choose the correct answer)?
 1. Technically, photons (via fiber optics) are easier to transfer than electrons, which are particles with mass.
 2. The possibilities of modes and superposition in the spin of electrons are more complex and will not allow the process.
 3. As we have seen in the spin appears in the formulas half the angle instead of the angle, which completely disrupts the simple options of 50-50.

Answer: Mathematically there is no difference between spin and polarization. Technically, with photons this is much easier, although they have not yet been able to apply the method over long distances.

2. Alice transferred 30,000 photons to Bob. Due to inaccuracies in the positioning of the measurement systems, there was a one-degree gap between Alice's measurement bases and those of Bob. That is, when Alice measured at the horizontal / vertical base she measured at an angle of θ and in the direction perpendicular to it (it does not matter if we call it x or y) that is, in my direction, and when she measured at the diagonal base she measured at an angle of θ and in the direction perpendicular to it. In contrast, Bob measured at the horizontal / vertical base at an angle θ and in the direction perpendicular to it, and at the diagonal base at an angle θ and in the direction perpendicular to it. Alice and Bob are unaware of the error. Beyond this error, there was no eavesdropping in this experiment.
- reminder:

$$|\theta\rangle = \cos \theta |x\rangle + \sin \theta |y\rangle$$

$$|\theta^\perp\rangle = \sin \theta |x\rangle - \cos \theta |y\rangle$$

- α. When Alice and Bob compared the measurement bases, they found that they measured the same base for 15,073 photons. Can this deviation (from the expected case of half of the photons) be explained by the inaccuracy in the measurement?
- β. Of the 15,073 relevant photons, Alice and Bob made an open comparison of 6,000 randomly selected photons. In how many photons will they discover that there is a difference between their results? Use Dirac markings to calculate the result.
- γ. Why in section B the question was about how many photons?
- δ. Will the deviation that Alice and Bob discover make them suspect the existence of an eavesdropper? Explain your answer.

Answers:

- α. There is no connection. This deviation is a statistical deviation from the expected 50% (15,000), and is not related to the measurement results (affected by the measurement errors), but to the chance of both to choose the same measurement basis.
- β. Consider one case (all other calculations are the same) - when Bob's situation is horizontal to his method i.e., what is the chance that with Alice the situation is vertical, i.e.. The overlap will be: $|1^\circ\rangle = \cos 1^\circ |x\rangle + \sin 1^\circ |y\rangle$

$$\cos 1^\circ \langle y|x\rangle + \sin 1^\circ \langle y|y\rangle = \sin 1^\circ = 0.0174$$

The probability will be (after squaring) $3.046 \cdot 10^{-4}$. That is, about one in 3,283. Out of 6,000 photons, there is likely to be a difference in two of them.

- γ. All of these are probabilistic calculations, and therefore inaccurate. We discovered that these are two photons, a very small number, and there can certainly be a deviation from it.
 - δ. Such a small deviation cannot be caused as a result of ignition. Normal spark plugs should cause a deviation of about a quarter of the photons tested, i.e. around 1,500 photons.
3. Alice sends a stream of photons to Lviv (pictured here from left to right).
- α. Does anyone eavesdrop? If so, bring evidence. If not, explain what the table would

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
The direction Alice was measuring	X	X	+	X	+	+	+	X	+	X	+	X	X	X	+	+	+	X	+	X	+	+	+	X
Alice's result	\	/		/	-			\	-	\	-	\	/	\			-	/		/	-		-	/
The direction in which Bob measured	+	X	+	+	+	X	+	+	X	X	+	+	X	+	X	+	+	+	+	X	X	X	+	+
Bob's result	-	/		-	-	/	-	-	\	\	-	-	\		/		-	-		/	/	\	-	

look like in case someone eavesdropped.

- ב. Find the code that will be received (or its different versions in case of eavesdropping).

Answers:

- א. Yes. It should be noted that in measurements 7,13 the results are different. Although it is probable that we would expect different measurements in about 3 cases, it is still very possible that there will be only two, since this is a small number of measurements.
- ב. The code is different because there are eavesdroppers. Alice's code is: 101001101011. Bob's code is: 101101001011

Quantum coin tossing

Follow the link below (Or search YouTube: "What is a quantum coin toss?"):

<https://www.youtube.com/watch?v=UjFkly1GTlk>

The coin toss can be illustrated with the help of the previous cards. Divide into two groups. One group chooses one of the colors, and measures regularly according to the same color. The second group randomly measures at both bases (the two colors), in each measurement it indicates to itself which color it chose, and what the result was. At the end of the process, the second group must guess what color the first group chose to test. If she's right in her guess - she's the winning team. How do you know what color the first group actually tested? The first group reads to the second group the whole series of unity and zeros. The second group is aware that in all measurements in one color the results are the same as its results, and the measurements in the other color are divided equally (roughly, the larger the number of measurements the more accurate it will be) between identical and different results.

Why can not be cheated? The first group must prove to the second that it was wrong in its guess (or admit that it was right). In contrast, the second group only needs to guess. That is, only the first group has an option to cheat, it seems that this is not possible. The first group does not know what color each measurement was made by the second group, and it conveys the results. She cannot aim for good results for her, because she does not know what good results are for her.

exercise

In one class, where a significant award was promised to the winning team, the first team cheated. The scam was planned as follows: In each of the notes, both sides were measured. The second group had a mole that reported to the first group what color each note was measured on. After the second group guessed a blue color, the first group passed a series of some unity and zeros. Thus proved that the guess of the other team was wrong, and thus won the game.

Questions:

1. How did the deceptive group build the series of unity and zeros it transmitted?
2. Is it possible to cheat in this way even with a real quantum coin toss?

To prevent such cheating it was decided that the other group would write its guess on a note and hand it over to the teacher. The note will be read only after passing the unity and zeros. The first group did not say desperate, and planted a mole in the second group, and thus knew in each and

every note what color it was measured. The mole was unable to convey to her senders her group's guess.

3. How will the cheating group organize the series of unity and zeros it reads? Will you succeed in her deception?

Answers:

1. The team read the Red Side results series.
2. In a true quantum coin toss it is not possible to measure on two different bases, so cheating is not possible.
3. The group will select in each measurement to read the result of the measurement in a color that matches the measurement of the other group. By doing so, she will see that she knows all the results in each of the colors. However, this will only reveal her disgrace, since in this way she will see that she has not measured on a single consistent basis.

Chapter 5 - Bosons and Fermions

We've talked before about the spin of the electron. The size of the spin of the electron is (and in each axis it can be in the positive or negative direction). In fact, every particle in nature has a spin. The size of the spin always comes in whole multiples of, i.e. the possible spin values for particles are. The particles can be sorted into two groups: particles with a whole spin value, i.e., in contrast there are particles with a half whole spin, i.e.. It turns out that whole-spin particles behave substantially differently from half-whole-spin particles. Particles with a whole spin are called bosons, after the Indian physicist Satindra Nat Boz, while particles with a whole spin are called fermions, after the Italian-American physicist Enrico Fermi. Among the fermions are particles such as the electron and the proton. An example of a boson is the photon. $\frac{\hbar}{2}, 0, \frac{\hbar}{2}, \hbar, \frac{3}{2}\hbar, 2\hbar, \frac{5}{2}\hbar, \dots$ $s = 0, \hbar, 2\hbar, 3\hbar, \dots$ $s = \frac{\hbar}{2}, \frac{3}{2}\hbar, \frac{5}{2}\hbar, \dots$

The main difference between bosons and fermions is in the law called Pauli's Prohibition Principle. This principle, formulated by physicist Wolfgang Pauli, states that two fermions cannot be in the same state. There must always be at least one physical size in which they have different values. This principle applies only to fermions (having a half-full spin) and not to bosons (having a full spin).

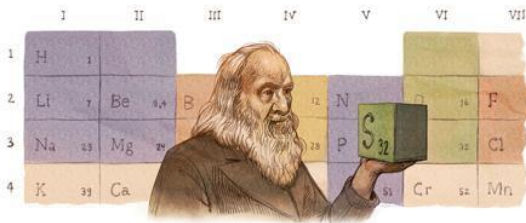
The photons are bosons (they have spin 1, meaning the size of their spin is), and therefore Pauli's prohibition does not apply to them. This is why we can produce a laser - a flux of photons that are identical to each other in all their properties. \hbar

This is different for electrons, which are, as mentioned, fermions. We will expand on this in the next chapter "Introduction to Chemistry".

Introduction to Chemistry

This topic includes many additions regarding the meanings of all physical sizes. This means that it has many parts that are technical and incomprehensible to students. At the same time it is an important subject in the applications of quantum physics. In light of this one should consider whether to include this or not, of course within the time frame possible.

One of the most important applications to the Pauli Prohibition Principle exists in the way populations of electrons in an atom. It is the manner in which the electrons are populated that gives each atom its chemical properties. In 1869 the Russian chemist Dmitry Mendeleev proposed the periodic table of chemical elements. Only a few decades later did they realize that the order in this table is in fact based on principles in quantum physics.



We have already seen that in related states (a state in which a particle has a limited area in which it can move) we get quantization - energy levels. So in the hydrogen atom and in the same case we called it a "particle in a box". In fact, despite the absurdity of it, quantization is not a quantum phenomenon, but a classical wave phenomenon. In any related wave,

such as a guitar string, there are discrete values of possible frequencies. In quantum physics every particle has wavy properties, so there are cases where discrete values are obtained, similar to any wave. When an electron rotates around the atomic nucleus it is in a bound state, so its physical magnitude values come at allowable levels. Due to the principle of uncertainty we are notCan talk simultaneously about all physical quantities (location, momentum, energy and more). However, there are four physical sizes that we can know simultaneously. These sizes are:

1. The energy of the electron.
2. The angular momentum of the electron.
3. Angular momentum component in any direction (we can not determine two different components simultaneously). The convention is to talk about a component of angular momentum.z
4. A component of the spin.z

Each of these sizes has a quantum number that characterizes it. The quantum number is an expression that the possible values come in discrete portions. We will now present the rules for determining the allowable quantum numbers, and for understanding the possible values for the physical quantities we have presented. The way to find the allowed values goes beyond our field of study, and is based on solving the Schrödinger equation, which requires high-level mathematics. We present here only the result.

1. energy. We know that the allowable energy levels in the Bohr model are when in the hydrogen atom, whereas it is the quantum number so that. Although the Bohr model is inaccurate (this is a classic semi-model), its energy levels are correct. Is called the main quantum number in this case. $-\frac{R^*}{n^2} = 13.6eV$ $n = 1, 2, 3, \dots n$
2. Angular momentum (hereinafter: TNZ). The normal momentum we know is also called linear momentum, and it is an expression of the velocity of a body (but also its mass) in the direction - some line (therefore called linear momentum). Similarly there is a physical quantity called angular momentum Quantum has certain allowable values of the angular momentum of the electron, and they also depend on the energy level at which the electron is. The possible angular momentum values are when it is the quantum number of the angular momentum. The allowable values depend on the main quantum number. And always exists i.e.:. Thus, if we are at the basic level then must exist, and in any case:

if we are at the first stimulus level then can be that, but also can be that and then $L = \sqrt{l(l+1)} \cdot \hbar$ $l = 0, 1, 2, \dots, (n-1)n = 1l = 0L = \sqrt{0 \cdot 1} \cdot \hbar = 0n = 2l = 0l = 1L = \sqrt{1 \cdot 2} \cdot \hbar = \sqrt{2}\hbar$.

3. The angular momentum component in the axis direction (also called: the projection of the TNA in a selected axis). Classically, if it is the magnitude of the angular momentum, then it can have any value in between. All values are possible, but only discrete values. The second amendment relates to the principle of uncertainty. We have seen before that it is not possible to determine the spin values in the various axes at the same time. The same can be said about the angular momentum. It is not possible to know simultaneously the values of the various components of the angular momentum. Therefore, if for example, then it follows that, that is, we know the values of the three components simultaneously, which is contrary to the principle of uncertainty $L_x - L_y = L_z = 0$ ¹⁷. Therefore, must exist. Component values of the permissible angular momentum are, when it is the relevant quantum number, and its permissible values are (or). Note that this is always the case (except in the case of) that $|L_z| < L$ $L_z = m\hbar$ $m = -l, -l+1, \dots, 0, 1, \dots, l$ $|m| \leq l$ $|L_z| < L$.
4. A component of the spin (also called: spin spin on a selected axis). We have already seen that if we measure the spin of the electron in the direction of any axis (and we refer as a convention to an axis) we can get two results. Thus we have here a quantum number that can have two values: $\pm \frac{\hbar}{2} m_s \pm \frac{1}{2}$.

It follows that the electron in the atom has a quantum state characterized by the four quantum numbers we have numbered. We can mark the situation as follows:. We would like to expand a little here on the atom model according to quantum mechanics - the Schrödinger atom model. This model is quantum, meaning it refers to the fact that an electron has a wave function that describes the probability of finding it in one place or another. In this model we are not talking about self-states of position or momentum, but as stated about states that are self-states of the four sizes: energy, L_x , L_y , L_z , spin. The wave function describes the values of each of the four sizes, and on the other hand describes the superposition of the position states. Just as the state is a self-state of spin up, and describes the probabilities of being in a right / left spin, so the state (or wave function) is a self-state of the said four physical quantities, and describes the probability of being in any position and position (it is a continuous function of infinite positions, not Connection of two / three possible locations). for example, For the situation, the function of the wave is, when it is some coefficient, is the distance from the nucleus, is the limited radius of Bohr (some correction to the atomic radius of Bohr). With the help of this function (much simpler than in cases where) we can find what the probability is of being in a particular

$$\text{location } |n, l, m, m_s\rangle | \uparrow \rangle = \frac{1}{\sqrt{2}} | \rightarrow \rangle + \frac{1}{\sqrt{2}} | \leftarrow \rangle \Psi_{n,l,m,m_s}(\vec{r}) n = 1 \Psi(r) = A \cdot e^{-\frac{r}{a_0}} A r a_0^* n > 1^{18}.$$

¹⁷ The exception is the case in it $L = 0$, where $L_x = L_y = L_z = 0$. We will not explain here why this case does not contradict the principle of uncertainty.

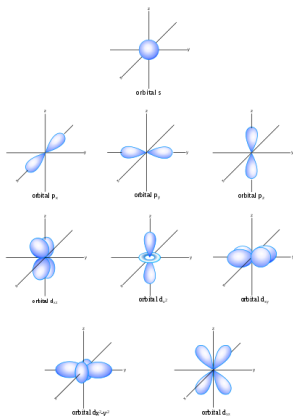
¹⁸It is important to emphasize that the calculation of the probability is not trivial, and requires an integral calculation of the square wave function in a particular domain. There is no significance to probability in a point location but in an area.

As stated, the electron in an atom has a quantum state characterized by the four quantum numbers we have counted. We can mark the situation as follows: The electron is free, so it is not possible for two electrons to be in the same state. This means that it is not possible to populate a large number of electrons at the basic level. We will try to see how many electrons can be populated at each level. $|n, l, m, m_s\rangle$

How many electrons can be populated at the basic level? This is a situation in which, and therefore commits, and in any case. The spin can have two values (this is a fact that does not vary from level to level). In total no more than two electrons can be populated at the basic level. $n = 1, l = 0, m = 0, m_s = \pm \frac{1}{2}$

We will move on to the next level - the first trigger level,. In this case it may be then, but it may be, then. A total of four cases, each of which creates two states because of values, so there are eight different states at the first trigger level. We will list them: $n = 2, l = 0, m = 0, l = 1, m = -1, 0, 1, m_s$

$$|n, l, m, m_s\rangle = |2, 0, 0, \frac{1}{2}\rangle, |2, 0, 0, -\frac{1}{2}\rangle, |2, 1, -1, \frac{1}{2}\rangle, |2, 1, -1, -\frac{1}{2}\rangle, |2, 1, 0, \frac{1}{2}\rangle, |2, 1, 0, -\frac{1}{2}\rangle, |2, 1, 1, \frac{1}{2}\rangle, |2, 1, 1, -\frac{1}{2}\rangle$$



A brief look at the periodic table reveals that indeed in the first row there are two elements, and in the second there are eight elements. Later it seems that this is directly due to the number of situations.

In fact, the energy level is accurate in the hydrogen atom only. The hydrogen atom has a single electron, so the Schrödinger equation can be solved for it in a relatively simple way. As more electron additions occur, the equation becomes more complex, and the solution of the hydrogen atom becomes less accurate. Moreover, due to various effects the energy becomes dependent on other quantities besides the value of n .

Every body always strives to be in a state of lower energy. It is an energetic observation of the aspiration of each body to fall to the ground, and of any mass attached to the spring to be drawn towards the state of equilibrium. Therefore, any electron we add to the atom will strive to be at the lowest energy level (if it is at a high level, it will drop a level and emit a photon). However, due to Pauli's Prohibition Act, he can only be in a situation that is not "occupied" already. This can be seen as an analogy for filling a glass with water. The water prefers to be at the bottom of the glass, but if the bottom of the glass is already "occupied", the water will start to fill the glass more and more at the top. We will now formulate the rules of preference of the electrons for populating the various states. These rules are called the Madlong Rules, after the German physicist Erwin Madlong who formulated the rules in 1936.

The first rule states that given two modes, the lower energy level is in a state where the value of the amount is lower. $M = n + l$

The second rule states that for two modes having the same, the energy level will be low in a state where its value is lower. Mn

There are other rules and exceptions, but we will ignore them in this framework.

We would like to use the usual markings for energy levels. Each level is called an atomic orbital. The orbitals are characterized by numbers, where the value of n is written as a number, while the value of l is denoted by letters in the following order: $s, p, d, f, g, h, i \dots$ when pronounced by the letter, the orbital is pronounced by, and so on. $n, l, m, s, p, d, f, g, h, i \dots l = 0, s, l = 1, p$

Regarding the orbitals we will note that the values of the quantum numbers define the dependence of the wave function of the electron on the angle (while values determine the dependence of the function at a distance from the center). The angular function (called a function) is often illustrated (spherical harmonics) using a balloon-like graph, which describes the probability (or rather the root of the probability) of being dependent on direction. This of course goes beyond the level taught here, but since we often meet with these paintings, we chose to mention them.

How many atoms can be inhabited in one orbital? Orbital is characterized by, how many different modes can be constructed in such a mode? As the number of possibilities for numbers. For a certain, there are options for values, after all. Each set of has of course two possible values (m_s). Therefore in each orbital electrons can be populated. n, l, m, s, l, m, m_s $2l + 1, m_s = -\frac{1}{2}, \dots, 0, 1, \dots, \frac{1}{2}$ $2 \cdot (2l + 1)$

In the hydrogen atom H has a single electron, so it will be inhabited at the lowest level. The helium atom has two electrons, and will also be populated at the same level. The notation for this population is that it means that there are two electrons in Orbital. $1s^2$

We will move on to the next atom - the lithium atom Li. This atom has three electrons. The first two are inhabited at a level, but there is no "room" for more electrons, so they now move to a state in which, that is, to a state in, which is marked. The total population is $1s^2 2s^1$

In general, for a state, i.e. for a state where there are 2 electrons that can be populated. For a state, i.e. a state in which electrons can be inhabited. For mode 10 can be populated, and for mode 14 electrons can be populated. $2l + 1 = 2 \cdot 3 = 6$

We will now try to see the population of electrons in the iron atom Fe, an atom with 26 electrons.

First, for, i.e. for, 2 electrons can be populated (the first two - as in the helium atom), and the population is $1s^2$.

We'll move on to the situation In it, that is, in a state in it, also in which 2 electrons can be populated and the population is. A total of 4 electrons. $1s^2 2s^2$

We'll move on to the situation at him. This state can be a state of, or a state of. As mentioned, the previous state of population has the lowest number, ie the state in which 6 electrons can be

populated, and the total population is 10 electrons. This population is that of the neon atom. $M = 3n = 2, l = 1n = 3, l = 0nn = 2, l = 11s^2 2s^2 2p^6$

The next option is as stated in which two more electrons can be populated - a total of 12 electrons, and the marking is. Many times this is written in short, with the marking in the square brackets being that of the neon, and generally marking the last noble gas that was. We will not detail their meaning here. We will just note that they arrive before the ascent to the next number, and this means that the electrons are populated to some degree in their entirety. They appear at the right end of each row, before moving on to the next row. $n = 3, l = 01s^2 2s^2 2p^6 3s^2 [Ne] 3s^2 n$

Similarly, the state of will consist of the states that will come first, and, so we can add another 8 electrons in the population written in short (the noble gas organization). In total we populated 20 electrons. $M = 4n = 3, l = 1n = 4, l = 01s^2 2s^2 2p^6 3s^2 3p^6 4s^2 [Ar] 4s^2$

We will now move on to the situation, which consists of the situations that will come first, that will come second, and that will come third. In this situation, 10 electrons can be populated, but we, in the case of iron, need to populate only 6. The population will be or in the abbreviated spelling $M = 5n = 3, l = 2n = 4, l = 1n = 5, l = 01s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^6 [Ar] 4s^2 3d^6$.

More details can be found, for example, in https://en.wikipedia.org/wiki/Electron_configuration

Electrical conductivity

The exact explanation for the electrical conduction (as opposed to the Drude model which is inaccurate) is quantum, and is related to the fact that the electrons are fermions, meaning that they cannot all be in the same physical state. In this context we will explain this only in general.

In a single atom there are discrete states of energy (this fact already exists in the Bohr model, and is also true in quantum physics in solving the Schrödinger equation). When talking about a solid matter, there are many atoms in it, so there is an overlap between the atoms and the permissible energy states. The allowable energy states are very many, so they become more and more dense for certain energies. A situation is created where there are ranges of energies in which there are many and dense states - almost a sequence, so this is a permissible energy range. Other ranges where there are no allowable energy levels will be forbidden ranges. This creates the structure of the stripes - a structure of permissible energy levels that looks like a crossing - stripes (sequence) of permissible energies, and between them gaps of forbidden energies.

Electrons are fermions, which means that not everyone can be in the same state - in the same energy. That is, if a particular electron "captures" some energy (and each particle strives to capture the lowest possible energy), then the next electron will be "forced" to receive a higher energy. This is somewhat reminiscent of water that fills a glass, the more water we add, they will not be able to be at the lowest altitude because it is occupied, but will reach a higher place - just as the water level rises as we add water, so does the energy level of electrons.

What sets a conductor apart from an insulator is that the number of electrons does not fill the allowable energy band, but only its portion. In a strip insulator full of electrons, and therefore even if we give them energy, they will not be able to reach the next strip, since there is an energy gap between the stripes (unless we give a very, very high voltage). In a conductor, on the other

hand, the electrons normally fill only part of the strip, so that any energy we give them can raise them to a higher energy in the strip (since the strip has a sequence). Raising energy means giving kinetic energy - the movement of electrons, that is, the existence of an electric current.